ELEMENTARY MATHEMATICS

PART I

ELEMENTARY MATHEMATICS

THIS course is divided into three parts:—

PART I deals with:—Revision of First Principles of Arithmetic — Extension of Arithmetical Processes Algebra—Geometry and Mensuration.

PART II deals with: Arithmetic related to Commerce — Algebra - Geometry and Mensuration related to Technology and leading to more advanced mathematical work.

PART III deals with:—Arithmetic related to Commerce—Algebra—Trigonometry —Geometry and Mensuration related to Technology.

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ELEMENTARY MATHEMATICS

SPECIALLY PREPARED FOR CENTRAL SCHOOLS, SENIOR ELEMENTARY SCHOOLS, AND UPPER STANDARDS (VI, VII, VIII)

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PART I

(*for Pupils of* 11–13 *years*)

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SECTION II

Extension of Arithmetical Processes

SECTION III

Algebra

SECTION IV

Geometry and Mensuration

TABLES

British Money

4 farthings	= 1 penny (1d.).	A florin	= 2 shillings.
12 pence	= 1 shilling (1/- or 1s.).	A double florin	= 4 shillings.
20 shillings	= 1 pound (£1).	A crown	= 5 shillings.
21 shillings	= 1 guinea (£1, 1s. 0d.).	Half a crown	= 2 shillings and 6 pence.

British Measure of Length

12 inches (ins.).	= 1 foot (ft.).	36 ins.	= 1 yd.
3 ft.	= 1 yard (yd.).	220 yds.	= 1 furlong.
22 yds.	= 1 chain (ch.).	1760 yds.	= 1 mile.
10 chs.	= 1 furlong.	80 chs.	= 1 mile.
8 furlongs	= 1 mile (ml.).	$5\frac{1}{2}$ yds.	= 1 rod; 1 pole; 1 perch.
		100 links	= 1 chain.
		40 poles	= 1 furlong.

British Square Measure

(12 ins. × 12 ins.) = 144 sq. ins. = 1 sq. ft.
(3 ft. × 3 ft.) = 9 sq. ft. = 1 sq. yd.
(22 yds. × 22 yds.) = 484 sq. yds. = 1 sq. ch.
10 sq. chs. = 4840 sq. yds. = 1 acre.
640 acres = 1 sq. ml.

British Cubic Measure

(12 ins. × 12 ins. × 12 ins.) = 1728 cu. ins. = 1 cu. ft.
(3 ft. × 3 ft. × 3 ft.) = 27 cu. ft. = 1 cu. yd.

British Measure of Capacity

4 gills = 1 pint.
2 pints = 1 quart.
4 quarts = 1 gallon.
2 galls. = 1 peck.
4 pecks = 1 bushel.
8 bushels = 1 quarter.
5 quarters = 1 load.
1 cu. ft. = $6\frac{1}{4}$ gallons, nearly.

British Measure of Mass

Avoirdupois Weight

16 drams	= 1 ounce (oz.).
16 ozs.	= 1 pound (lb.).
14 lbs.	= 1 stone (st.).
28 lbs.	= 1 quarter (qr.).
4 qrs. or 112 lbs.	= 1 hundredweight (cwt.).
20 cwts. or 2240 lbs.	= 1 ton.

Time Measure

60 seconds (secs.)	= 1 minute (min.).
60 mins.	= 1 hour (hr.).
24 hrs.	= 1 day.
7 days	= 1 week (wk.).
4 wks.	= 1 lunar month.
28 days	= 1 lunar month.
365 days	= 1 year (yr.).
52 wks. and 1 day	= 1 year.
12 calendar months	= 1 year.
366 days	= 1 leap year.
100 yrs.	= 1 century.

All the Metric Tables are included in the following table :—

1000	100	10	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
Kilo-metre.	Hekto-metre.	Deka-metre.	Metre.	deci-metre.	centi-metre.	milli-metre.
Kilo-litre.	Hekto-litre.	Deka-litre.	Litre.	deci-litre.	centi-litre.	milli-litre.
Kilo-gram.	Hekto-gram.	Deka-gram.	Gram.	deci-gram.	centi-gram.	milli-gram.

Elementary Mathematics

SECTION I

REVISION OF FIRST PRINCIPLES OF ARITHMETIC

Numeration and Notation

EXERCISE 1

1. Write in figures: five thousand and ten; forty thousand and eleven; one thousand one hundred; twenty thousand and ninety; three hundred thousand four hundred and five; sixty-seven thousand and eight; one million and a half; a quarter of a million.

2. Write in words: 3,070; 42,990; 510,078; 6,035; 110,204; 90,080.

3. Consider the number 30,933. Write down the values of the first 3, the second 3, and the third 3. Find their sum.

4. What is the difference between the values represented by the 8 and 9 in the number 1897?

5. Place a cipher somewhere between the first figure and the last in the number 57638, (i) to increase it as much as possible, and again (ii) to increase it as little as possible. Find the difference between these two numbers.

6. Write five numbers (each containing one significant figure only) which compose the number 75,328.

7. (*a*) How many thousands make a million?

(*b*) How many hundreds comprise a million?

(*c*) How often must 10 be written in a continued multiplication to produce a million?

8. The first odd number is 1, the second is 3, the third is 5, and so on. Write the fiftieth odd number.

9. Find the sum of the five consecutive numbers beginning at 85, and then find the average of them.

10. Find, without actual working, the average of :

(*a*) The five consecutive numbers beginning at 33.
(*b*) The seven ,, ,, ,, 29.
(*c*) The nine ,, ,, ,, 71.

Write a rule about finding the average of an odd number of consecutive numbers.

The Four Rules

Exercise 2

1. Add horizontally :

(*a*) 7356, 584129, 3687, 908, 7358, 69, 124768.
(*b*) 384689, 7831960, 459836, 178954.

2. The digits 7, 8, 9 can be arranged to form six different numbers, thus, 789, 978, 879, and so on. Form these six different numbers and then find their sum.

3. How much is fifty thousand eight hundred and seven short of a million ?

4. The population of a town ten years ago was one hundred and thirty-seven thousand and seventy, but now it is two hundred and nine thousand and thirty-six. Has the population increased or decreased ? By how many ?

5. Arrange the following digits : (i) to make the largest possible number, and (ii) to make the smallest possible number : 1, 3, 5, 7, 2. Then, find the difference between the two numbers.

Learn to multiply as shown below.

```
          3769 . . . Multiplicand.
           562 . . . Multiplier.
        ------
       18845   =3769 × 500
        22614  =3769 × 60
          7538=3769 × 2
        ------
Product=2118178=3769 × 562
```

Exercise 3

1. When 258420 is divided by x, the quotient is 365. Find the value of x.

2. Divide 275000 by (i) 19, (ii) 74, (iii) 749.

3. Find the least number that must be added to twenty thousand apples so that they may be divided into 58 equal lots.

4. (*a*) 764928×760 ; 209 ; 385.
(*b*) 58769×298 ; 370 ; 65.
(*c*) 90876×1095 ; 483 ; 276.

Give the answer to the exercises in Question 5 to the nearest whole number. Thus: if the remainder is *half or greater than half* of the divisor, the quotient must be increased by one; but, if the remainder is *less than half* of the divisor, the quotient remains unchanged.

5. (*a*) $7643012 \div 19$; 769 ; 593.
(*b*) $5980356 \div 37$; 1196 ; 970.
(*c*) $1843957 \div 290$; 368 ; 17698.

6. If the quotient is 7309, the remainder 17, and the dividend 285068, find the divisor.

Short Methods

Note these Examples.

$$73 \times 25 = 73 \times \frac{100}{4} = \frac{7300}{4} = 1825$$

$$58 \times 125 = 58 \times \frac{1000}{8} = \frac{58000}{8} = 7250$$

$$\begin{aligned} 764 \times 99 &= (764 \times 100) - (764 \times 1) \\ &= 76400 - 764 = 75636 \\ 569 \times 763 &= 569 \times 700 + 569 \times 7 \times 9 \\ &= 398300 + 35847 \\ &= 434147. \end{aligned}$$

Find the shortest method of working the following and write the results.

EXERCISE 4

1.	Multiply 1795 by	(i) 25,	(ii) 125,	(iii) 2500.
2.	,, 3563 by	(i) 25,	(ii) 125,	(iii) 2500.
3.	,, 7935 by	(i) 99,	(ii) 98,	(iii) 199.
4.	,, 3357 by	(i) 963,	(ii) 549,	(iii) 327.
5.	Divide 67843 by	(i) 10,	(ii) 100,	(iii) 1000.

Divisibility of Numbers

Any number is divisible, without remainder,

By 2, if the last digit is even or if it is a cipher.

By 4, if the last two digits form a number of which 4 is a factor, or if they are two ciphers.

By 8, if the last three digits form a number of which 8 is a factor, or if they are three ciphers.

By 3, if the sum of the digits is a number of which 3 is a factor.

By 6, if the last digit is even, and the sum of the digits is a number of which 3 is a factor.

By 9, if the sum of the digits is a number of which 9 is a factor.

By 5, if the last digit is 5 or a cipher.

By 10, if the last digit is a cipher.

By 11, if the sum of the alternate digits equals the sum of the remaining digits, or if the difference of these sums is a product of 11.

Roman Numerals

1	I	11	XI	25	XXV	200	CC
2	II	12	XII	30	XXX	300	CCC
3	III	13	XIII	35	XXXV	400	CD
4	IV	14	XIV	40	XL	500	D
5	V	15	XV	50	L	600	DC
6	VI	16	XVI	60	LX	700	DCC
7	VII	17	XVII	70	LXX	800	DCCC
8	VIII	18	XVIII	80	LXXX	900	CM
9	IX	19	XIX	90	XC	1000	M
10	X	20	XX	100	C	2000	MM

Exercise 5

1. Select from the following numbers those divisible by (i) 2, (ii) 3, (iii) 4, (iv) 5, (v) 6, (vi) 8, (vii) 9, (viii) 10, (ix) 11 :— 5673, 84951, 3582, 12348, 7290, 365, 33000, 18432095.

2. Write the Roman numerals for 9, 28, 65, 48, 105, 550, 87, 93, 1890, 1926.

3. Write in Arabic numerals: CXXI, MCMXXV, LIX, XLIV, XC, CCXC, CDLXXXIV.

4. Write the present year in Roman numerals.

British Money

Exercise 6

1. Find how much the sum of the following amounts is short of ten thousand pounds: £17, 10s. 11d.; £183, 15s. 9d.; £7843, 9s. 11d.; £115, 19s. 9d.; £777, 17s. 7d.

2. On a certain day tin was sold at £110 per ton. If a merchant bought at this price and sold again at £147, 6s. 8d. per ton, find his gain per lb. (2240 lbs.=1 ton).

3. £37, 16s. 8¾d. × 35, 109, 267.

Give the answers to Question 4 correct to the nearest penny. Thus: do not work beyond the pence, and if, after dividing the pence, the remainder is *half or greater than half* of the divisor, the pence part of the quotient must be increased by one; but if, after dividing the pence, the remainder is *less than one half* of the divisor, the quotient remains unchanged.

4. (i) £7896, 14s. 11d. ÷ 19, 59, 138.
(ii) £3784, 15s. 6d. ÷ 52, 28, 365.
(iii) £776, 19s. 6d. ÷ 12, 129, 3059.

5. A dealer bought 500 coats at 31s. 6d. each, and sold them for £950, 12s. 0d. Find *to the nearest penny* the profit on each coat.

6. Find the difference in £ s. d. between the greatest and least of the following: 1938 threepenny pieces, 35 guineas, 197 half-crowns.

7. When butter is selling at 1s. 5½d. a pound, how many pounds can be bought for £8, 15s. 0d.?

8. A greengrocer bought 2400 oranges for £3, 7s. 6d. If he threw away 500 of them and sold the remainder at 5 for 2d., find his profit or loss.

9. Find to the nearest penny the average price per pound weight of rubber, when 32,000 pounds weight are sold for £4020, 6s. 8d.

10. When 201,000 cwts. of wheat are sold for £84,876, find to the nearest penny the average cost of one hundredweight.

Length

The British Unit of Length is the Yard. The Standard Yard is kept at the Offices of the Board of Trade in London; it is the distance between two lines marked on a bronze bar, when at a temperature of 62° F. Lengths are measured in various units, *e.g.* a line may be measured in inches, a longer length in feet or yards; land may be measured in chains, ocean depths in fathoms (6 ft. = 1 fathom), very long distances in miles and furlongs.

Contractions. 4′ 6″ = 4 feet 6 inches = 4 ft. 6 ins.

Exercise 7

1. Estimate the length of your class-room, playground, or hall. Measure and check your estimate.

2. How many revolutions will a wheel make in travelling 15 miles 7 furlongs 5 chains if the circumference of the wheel is 10 ft. 7½ ins.?

3. How many lengths of tape, each $8\frac{1}{4}$ ins., can be cut from a roll half a mile long ? What length of tape would remain ?

4. Calculate, by measurements from a map, the shortest distance from :—

(i) London to Newcastle ;
(ii) Leeds to Birmingham ;
(iii) Liverpool to Manchester.

[Use thread, or dividers, and make calculations from the scale given on the map.]

5. A cricket-field is 100 yds. long and 90 yds. wide. Find the length (in miles, furlongs, and yards) of wire needed to go round the field ten times.

6. A field is $72\frac{1}{2}$ yds. long and 35 yds. wide. How many hurdles, each 4 yds. 1 ft. 6 ins. long, will be required to fence all round it ?

The Metric Unit of Length is the Metre. The Standard Metre is kept in Paris ; it is the distance between two lines marked on a platinum bar, when at a temperature of 0° C.

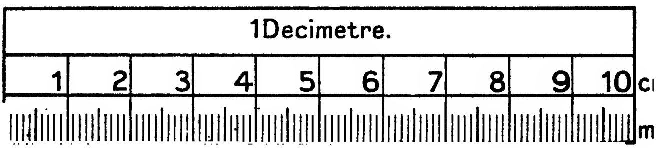

FIG. 1.

The Metric System is used by scientists all over the world, and is in common use in almost every country except the British Empire.

It has many advantages, *e.g.* :

1. Reduction is made easy, because we have always to multiply or divide by 10, 100, and so on.

2. The units of length, area, volume, and weight bear a definite relation to one another.

3. The naming of the various multiples or divisions of the unit is common to all the tables.

Examine a metre rule. Find on it the length of a decimetre, which $=10$ centimetres $=100$ millimetres, as shown in fig. 1.

Learn these Greek prefixes : Deko $=10$ times ; Hekto $=100$ times ; Kilo $=1000$ times.

Then, 1 Dekametre = 10 metres ; 1 Hektometre = 100 metres ; 1 Kilometre = 1000 metres.

Now learn these Latin prefixes : deci = $\frac{1}{10}$; centi = $\frac{1}{100}$; milli = $\frac{1}{1000}$.

Then, 1 decimetre = $\frac{1}{10}$ metre ; 1 centimetre = $\frac{1}{100}$ metre ; 1 millimetre = $\frac{1}{1000}$ metre.

As these are long words they are contracted thus : m = metre or metres ; K = Kilo ; H = Hekto ; D = Deka ; d = deci ; c = centi ; m = milli.

Therefore Km., Hm., Dm., dm., cm., mm. are easily understood.

A metre = 39·37 inches, very nearly.

Exercise 8

1. Draw lines : 5·1 cm., 4·9 cm., 3·5 cm., 6·3 cm.

2. How many mm. are there in 5 m. + 56 cm. + 9 mm. ?

3. Find by measurement the total length of three lines each 5·6 cm. in length.

4. Draw on squared paper a rectangle 10 cm. long and 5·2 cm. broad and then find its perimeter.

5. Draw a line 4 ins. long. Measure it in cm. What is the value of 1 in. in cm. ?

6. Draw a line 10 cm. long. Measure it in inches and decimals of an inch. What is the value of 1 cm. in inches ?

Area

Exercise 9

1. Draw on squared paper a square having a base of 12 divisions. How many small squares does the large square contain ?

2. Draw a square on any base and find its area by dividing it into smaller squares.

Area of a square = base × altitude.
= a side × a side.
= **(a side)^2**.

3. Draw on squared paper a rectangle 10 divisions long and 8 divisions wide. How many small squares does the rectangle contain ?

4. Draw a rectangle on any base and find its area by dividing it into squares.

Area of a rectangle = base × altitude.

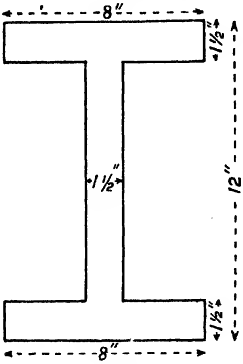

Fig. 2.

5. Find the area of the four walls of a room 25 ft. long, 24 ft. wide, and 10 ft. high.

6. Find the area of the section of a girder shown in fig. 2.

Area is the measure of the surface of a figure. *The Unit of Area* is the surface of a square of unit length. Thus in the British System of measurement we have a square inch, a square foot, etc., and in the Metric System we have a square centimetre, a square metre, etc. Areas are measured in square measure, *i.e.* the magnitude of an area is expressed by the number of squares of a definite size which would cover it.

Exercise 10

1. The area of a platform is 60 sq. yds. How many persons will it accommodate if each person requires $6\frac{1}{4}$ sq. ft.?

2. A book contains 96 sheets, each having an area of 40 sq. ins. Find in square feet the area of paper used in making the book.

3. A floor has an area of 21 sq. yds., and on it is a carpet of area 16 sq. yds. Find the cost of polishing the uncovered portion at $5\frac{1}{2}$d. per square foot.

4. The pressure of air is 15 lbs. per square inch. What is the total pressure on the surface of a kite having an area of $7\frac{1}{2}$ sq. ft.? Give the answer in lbs.

5. A plot of land has an area of 50 sq. m. Find its cost at 2d. per sq. dm.

6. Draw a square decimetre on squared paper and then show by ruling that it contains 100 sq. cm.

Volume

If we examine a cube we see that the edge has length only. the face has length and breadth, but the solid has length, breadth, and thickness.

Note —(1) A face is bounded by lines; (2) a solid is bounded by surfaces.

Exercise 11

1. Make a drawing of a square foot on a drawing board and rule it into 144 sq. ins.

2. Write down how many inch cubes would be required to cover this area.

3. How many times must this be done to build up one cubic foot ?

4. Assuming that one cubic foot is represented by one cubic inch, show how many cubic feet would be required to build up a cubic yard ?

5. Find the volume of the air in a class-room which is 8 yds. by 7 yds. by 7 yds. Give the answer in cubic feet.

6. Find the volume of the air in an attaché case 18 ins. long, 12 ins. wide, and 4 ins. high. Give the answer as a fraction of a cubic foot.

Volume is the measure of space occupied by a body.

The unit is the volume of a cube having each side of unit length. Thus, in the British System of measurement we have a cubic inch, a cubic foot, etc., and in the Metric System a cubic centimetre, a cubic decimetre, etc.

Exercise 12

1. How many cubic inches are there in 5 cu. yds., 12 cu. ft., 864 cu. ins. ?

2. Express 100,000 cu. ins. as cubic yards, cubic feet, and cubic inches.

3. Make dimensioned drawings of a cubic inch, a cubic centimetre, and a cubic decimetre.

4. Find the total volume of a stack of 8500 tiles, if each tile has a volume of 8 cu. ins.

5. If the volume of a brick is $121\frac{1}{2}$ cu. ins., how many bricks will make a stack having a volume of 1 cu. yd. ?

Capacity

Capacity refers to the power of holding. Our utensils for holding are named according to their internal volume. *The British unit of capacity* is the Gallon, the volume of which is 10 lbs. of pure water at 62° F. Other units are the quart, pint, etc.

The Metric unit of capacity is the Litre; it is the volume of a cubic decimetre, *i.e.* 1000 cubic cm. (written 1000 c.c.).

EXERCISE 13

1. A bucket holds 4 galls. 1 pt., and is filled from a water-tap in 1 min. How much water is discharged from the tap in 2 hrs.?

2. A tank weighs 92½ lbs. Find its weight in lbs. when it is holding 8 galls. 2 qts. of water.

3. A corn bin holds 5 bushels of corn. How long will it serve to feed 20 hens if each has a daily ration of 1 gill?

4. How often must a 5 c.l. bottle be used to fill a ½ litre flask?

5. Examine the following: (i) a 50 c.c. burette, (ii) a 20 c.c. pipette, (iii) a 250 c.c. graduated cylinder, and say what fraction of a litre is the capacity of each.

6. Place 10 c.c. of mercury in a ½ litre flask. How much water is still required to fill the flask?

Weight

The amount of matter in a substance is its mass. The measure of the mass of a body is its weight. *The British unit of weight* is the Pound. The Standard Pound is the weight of a certain lump of platinum kept by the Board of Trade.

The Metric unit of weight is the Gram (written 1 g.). It is the weight of one cubic centimetre (1 c.c.) of water at a temperature of 4° C.

EXERCISE 14

1. There are 28 boys in a class; 6 weigh 4 st. 7½ lbs. each, 8 weigh 3 st. 11½ lbs. each, 10 weigh ¾ cwt. each, and the remainder weigh 70¾ lbs. each. Find the average weight.

2. How many truck loads, each weighing 4 tons 15 cwts. 3 qrs. 21 lbs., can be taken away from 1000 tons of coal? What weight of coal would be left?

3. Find the total weight of 20 loads of metal, if each load weighs 5 tons 15 cwts. 84 lbs.

4. How many centigram weights are needed to balance a piece of wood weighing 2 Dg. 4 g. 3 dg. ?

5. (*a*) Draw a line one decimetre in length.

(*b*) Draw on squared paper a square decimetre.

(*c*) Make a cubic decimetre with paper or cardboard.

(*d*) Pour a cubic decimetre of sawdust into a litre jug. What do you notice ?

(*e*) Counterbalance a cubic decimetre of water and a kilogram. What can you say about their weights ?

6. Which is heavier, and by how many lbs.—1 ton or 1000 kgs. ? Assume that 1 kg.$=2\cdot2$ lbs. or $2\frac{1}{5}$ lbs.

Time

Time is the measure of duration. *The unit of time* is the Day. We may say that a day is the time the earth takes to rotate once completely on its axis.

The length of a year may be regarded as the time taken for a complete revolution of the earth round the sun, and is 365 days 5 hrs. 48 mins. 48 secs.

The normal year is divided into 12 Calendar Months—January, February, etc. A Lunar Month may be said to be the time the moon takes to make one complete revolution round the earth, and is 28 days, nearly.

A leap year$=366$ days. The number of a Leap Year is exactly divisible by 4, *e.g.* 1928 is a leap year; it is exactly divisible by 4, for $1928\div4=482$. But when a year is the last year of a century it is not a Leap Year unless its number is exactly divisible by 400, *e.g.* the year 1900 was not a Leap Year, but the year 2000 will be.

Contractions.—a.m. = ante-meridiem = before mid-day.
p.m. = post-meridiem = after mid-day.
B.C. = before Christ.
A.D. = Anno Domini = in the year of the Lord.

Exercise 15

1. Draw a clock face and show the position of the hands at 4.20 a.m. or p.m., as correctly as you can.

2. Write the exact date of your birth. How many days have you lived ?

3. Find from a railway time-table the shortest time it would take you to go from your home to London, Liverpool, Birmingham, Manchester, Glasgow, Edinburgh, Leeds, Bradford, and Preston.

4. What is the shortest time in which you can run 100 yds. ?

5. A train starting from Aberdeen at 5.15 a.m. arrives in London at 4.35 p.m. on the same day. How many minutes elapse on the journey ?

Factors and Multiples of Whole Numbers

A Multiple is a number which contains another number an exact number of times, *e.g.* 42 is a multiple of 7, 2, and 3.

A Prime Number is one that has no factors other than itself and unity, *e.g.* 3, 5, 7, 13.

Two numbers are said to be *prime* to one another if they have no common factor besides unity, *e.g.* 17 and 18 are prime to one another. When a number is contained an exact number of times in each of two or more other numbers it is said to be a *common factor* of the numbers, *e.g.* 7 is a factor of 35 and of 56. It is therefore a common factor of 35 and 56.

The highest common factor is written H.C.F.

EXERCISE 16

1. Find the H.C.F. of (*a*) 342 and 306, (*b*) 1210 and 550, (*c*) 361 and 114, (*d*) 5001 and 5256, (*e*) 39, 260, and 169, (*f*) 49, 1197, and 567.

2. Find the prime factors of 30,030.

3. Find the greatest number by which 2383 and 1771 can be divided so as to have a remainder of 3 in each case.

4. Find the prime factors of : 28, 220, 570, 460, 3610.

5. Find the common prime factors of (i) 36 and 60, (ii) 35 and 50, (*c*) 77, 21, and 63.

6. Find the highest number that will divide 3931 and 2621 and leave a remainder of 1 in both cases.

7. Find the length of the longest piece of rope that will exactly measure both the length and breadth of a field which is 105 yds. long and 120 yds. wide.

8. Find the largest sum of money that can be taken an exact number of times from £2, 3s. 6d. and £3, 4s. 6d.

Least Common Multiple

When a number contains each of two or more numbers an exact number of times it is known as a common multiple of these numbers, *e.g.* 70 is a common multiple of 2, 5, 7, and 10. The least common multiple is written L.C.M.

Note.—We can often determine L.C.M. by inspection by choosing a multiple of the greatest number, *e.g.* L.C.M. of 2, 4, 6, $8 = 8 \times 3 = 24$.

Exercise 17

1. Find the least quantity of flour that can be made up into bags of 3 lbs., 8 lbs., 20 lbs., 36 lbs., or 100 lbs.

2. What is the least number that can be divided by 4, 5, 6, 7, and each time leave a remainder of 3 ?

3. Three bells begin to toll at the same moment. They toll at intervals of 6 secs., 10 secs., and 24 secs. respectively. How long will it be before they all toll together again ?

4. Find the least amount of money required to buy either an exact number of bags at 12s. 6d. each, or an exact number of bags at 10s. 6d. each.

5. With a certain amount of money a man finds he can give an exact number of presents worth either 1s. 3d., 2s. 4d., 3s. 0d., or 7s. 0d. Find the least amount of money he must possess.

6. Change the fractions $\frac{3}{7}$ and $\frac{9}{11}$ into equivalent fractions having numerators of 27.

7. Change the fractions $\frac{4}{5}$ and $\frac{7}{8}$ into equivalent fractions having denominators of 80.

8. Reduce the following fractions to their lowest terms either by taking out the common factors, by inspection, or by finding the H.C.F. :—

(*a*) $\frac{75}{325}$, $\frac{77}{121}$, $\frac{935}{2000}$, $\frac{171}{570}$, $\frac{230}{276}$, $\frac{236}{531}$.

(*b*) $\frac{720}{1881}$, $\frac{1230}{1070}$, $\frac{555}{3700}$, $\frac{411}{3333}$, $\frac{226}{791}$, $\frac{391}{1700}$.

Vulgar Fractions

A Fraction is one or more of the equal parts into which one whole is divided.

A Vulgar Fraction is a fraction expressed by two numbers placed one above the other and having a line between them, as $\frac{5}{8}$; the 8 is known as the denominator and the 5 as the numerator.

The process of striking out factors common to the numerator and denominator of a fraction is known as *cancelling*, and when this has been done the fraction is said to be in its *lowest terms*, *e.g.* $\frac{36}{96}=\frac{2\times2\times3\times3}{2\times2\times3\times8}$, or $\frac{12\times3}{12\times8}$. We may therefore express $\frac{36}{96}$ as $\frac{3}{8}$, thus cancelling by 12.

Addition and Subtraction

EXERCISE 18

1. Find graphically the difference between

(i) $\frac{5}{6}$ and $\frac{7}{9}$, (ii) $\frac{7}{11}$ and $\frac{5}{8}$, (iii) $\frac{9}{16}$ and $\frac{12}{45}$, (iv) $\frac{7}{12}$ and $\frac{8}{15}$.

2. Arrange the following fractions in ascending order of magnitude :—

(i) $\frac{5}{6}$, $\frac{3}{5}$, $\frac{4}{9}$, $\frac{7}{10}$; (ii) $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{8}$; (iii) $\frac{3}{5}$, $\frac{3}{7}$, $\frac{19}{35}$, $\frac{51}{70}$.

3. Arrange the following fractions in descending order of magnitude :—

(i) $\frac{5}{8}$, $\frac{3}{12}$, $\frac{4}{24}$, $\frac{3}{10}$; (ii) $\frac{7}{10}$, $\frac{2}{5}$, $\frac{11}{20}$, $\frac{13}{30}$; (iii) $\frac{1}{3}$, $\frac{1}{8}$, $\frac{3}{16}$, $\frac{15}{32}$.

4. Find the value of

(i) $\frac{5}{8}+\frac{3}{4}+\frac{7}{16}+\frac{2}{11}$; (ii) $6\frac{1}{2}+7\frac{3}{4}+9\frac{1}{5}+6\frac{6}{7}$;

(iii) $\frac{5}{2\times3}+\frac{6}{2\times4}+\frac{7}{3\times4}+\frac{8}{4\times5}$; (iv) $\frac{3}{5\times2}+\frac{4}{5\times5}+\frac{9}{2\times2}+\frac{7}{2\times3}$.

5. Find the value of

(i) $3\frac{1}{4}-\frac{7}{8}$; (iv) $7\frac{1}{5}-5\frac{9}{10}$.
(ii) $15\frac{2}{5}-1\frac{7}{10}$; (v) $15\frac{3}{7}-8\frac{2}{9}$.
(iii) $8\frac{1}{4}-3\frac{3}{15}$; (vi) $18\frac{11}{24}-16\frac{17}{32}$.

6. From a roll containing $50\frac{5}{8}$ yds. a salesman cuts $25\frac{5}{6}$ yds. What length remains ?

7. A racecourse is $100\frac{5}{8}$ yds. long. When the winner of a race touches the tape the next man has run $98\frac{3}{7}$ yds. By how much was the race won ?

8. Find the sum of these measurements : $\frac{11}{12}$ ins., $3\frac{1}{4}$ ins., $2\frac{1}{16}$ ins., $4\frac{3}{10}$ ins., and $1\frac{5}{9}$ in.

9. Find the value of £$1\frac{2}{3}$+£$2\frac{1}{12}$+£$3\frac{1}{6}$+£$1\frac{1}{16}$.

10. (*a*) Find the perimeter of a rectangle $4\frac{2}{3}$ ins. long and $2\frac{3}{8}$ ins. wide.

(*b*) By how much does the length exceed the breadth ?

11. I spent $\frac{5}{9}$ of my money on food, $\frac{1}{6}$ on clothing, and $\frac{1}{8}$ on gifts. What fraction of my money is left ?

12. Which is greater, $\frac{1}{8}$ of 11 or $\frac{1}{7}$ of 12? By how much?

13. If $\frac{1}{8}$ of my money amounts to £1, 8s. 6d., how much money have I altogether?

14. After spending $\frac{3}{8}$ of my money I find that I have £5, 5s. 0d. left. How much had I at first?

15. Three-quarters of my money is greater than three-fifths of my money by 30s. How much money have I?

16. A tap can fill a bath in ten minutes. Another tap can fill the same bath in a quarter of an hour. How long would it take both taps running together to fill the bath?

17. When $\frac{3}{5}$ of a class are present there are 12 absent. How many pupils are there in the class?

18. I gave $\frac{2}{9}$ of my money to Ben, and $\frac{5}{12}$ of it to Sam, and then I had 2s. 2d. left. How much money had I at first?

19. I gave $\frac{5}{11}$ of a sum of money to a boy and $\frac{3}{7}$ of the same sum to a girl. If the difference between the amounts each received was 3s. 6d., find the sum of money I had at first.

20. A pole is driven into a river bed. If $\frac{1}{8}$ of its length is in the mud, $\frac{2}{5}$ in the water, and $8\frac{1}{2}$ ft. above the level of the water, find the total length of the pole.

Exercise 19

Find the value of the following :—

1. $3\frac{1}{4}-2\frac{1}{7}+5\frac{3}{5}-3\frac{3}{14}$.

2. $5\frac{1}{5}+3\frac{2}{5}-1\frac{1}{10}-4\frac{9}{16}$.

3. $7\frac{41}{77}-3\frac{1}{5}-2\frac{1}{11}+1\frac{6}{7}$.

4. Find the value of $3\frac{1}{3}$ ft. $+2\frac{1}{2}$ ft. $-5\frac{5}{8}$ ft. $+2\frac{1}{4}$ ft. $-1\frac{7}{16}$ ft.

5. Find the value of £$2\frac{3}{5}+$£$1\frac{5}{8}-$£$1\frac{5}{16}-$£$1\frac{3}{5}$.

6. A, B, and C own an estate. A owns $\frac{2}{3}$, B owns $\frac{1}{5}$, and C owns 10 acres. Find (i) the size of the whole estate; (ii) how many acres A owns.

7. Find the value of $\frac{7}{8}$ in. $-\frac{3}{8}$ in. $+1\frac{3}{16}$ in. $-\frac{1}{16}$ in.

8. Find the value of $2\frac{7}{8}$ ozs. $-4\frac{5}{8}$ ozs. $+5\frac{5}{16}$ ozs.

Multiplication and Division

Exercise 20

1. Simplify (i) $\frac{5}{11}$ of $3\frac{5}{6}$ of $5\frac{1}{2}$ of $\frac{23}{125}$.

(ii) $\frac{9}{13}$ of $1\frac{4}{9}$ of $\frac{16}{25}$ of $7\frac{1}{2}$ of 90.

(iii) $\frac{4}{9}$ of $3\frac{1}{3}$ of $\frac{27}{64}$ of $92\frac{1}{2}$.

2. Simplify (i) $9\frac{1}{2}\div3\frac{1}{3}$, (ii) $5\frac{1}{3}\div2\frac{7}{8}$; (iii) $6\frac{1}{4}\div5\frac{1}{25}$.

3. Simplify (i) $(\frac{5}{19}$ of $\frac{37}{115})+\frac{11}{23}$; (iv) $(\frac{9}{10}-\frac{3}{7})$ of $16\frac{1}{3}$

(ii) $(\frac{3}{11}$ of $\frac{77}{120})-\frac{1}{120}$; (v) $(\frac{5}{11}+\frac{4}{5})\div4\frac{1}{11}$.

(iii) $(\frac{5}{19}$ of $7\frac{3}{5})\div\frac{3}{5}$; (vi) $(\frac{17}{120}+19\frac{1}{3})\div5\frac{11}{120}$.

4. What quantity must be added to $\frac{5}{11}$ of $33\frac{1}{3}$ to make 20 ?

5. After spending $\frac{3}{5}$ of $\frac{10}{27}$ of half a crown, what part of half a sovereign is left ?

6. A strip of steel is $11\frac{1}{2}$ ins. long. How many pieces, each $\frac{3}{4}$ in. in length, may be cut from it if $\frac{1}{16}$ in. is wasted for each piece sawn off ? What is the length of the portion remaining ?

7. Three cwts. of soap are wrapped up in parcels each containing $1\frac{1}{3}$ lbs. How many parcels are there ?

8. The total weight of seven boxes is $53\frac{3}{7}$ lbs. Two of them weigh $3\frac{5}{8}$ lbs. each, three of them weigh $6\frac{5}{7}$ lbs. each, and another weighs $10\frac{1}{2}$ lbs. Find the weight of the seventh box.

9. $\frac{5}{11}+\frac{1}{19}+3+x=50\frac{1}{19}$. Find the value of x.

10. Divide the difference between $\frac{5}{24}$ and $\frac{19}{20}$ by their sum.

Simplification

Examples. (i) $(5\frac{1}{9}-1\frac{5}{6})\div(3\frac{1}{2}+5\frac{3}{8})$

(ii) $(\frac{6}{7}$ of $\frac{5}{16})+(5\frac{8}{11}\div\frac{9}{22})$

(i)

$$(5\tfrac{1}{9}-1\tfrac{5}{6})\div(3\tfrac{1}{2}+5\tfrac{3}{8})$$
$$=\left(3+\frac{18+2-15}{18}\right)\div\left(8+\frac{4+3}{8}\right)$$
$$=3\tfrac{5}{18}\div8\tfrac{7}{8}$$
$$=\tfrac{59}{18}\div\tfrac{71}{8}$$
$$=\frac{59}{\cancel{18}_{\,9}}\times\frac{\cancel{8}^{\,4}}{71}$$
$$=\tfrac{236}{639}$$

(ii)

$$\left(\frac{\cancel{6}^{\,3}}{7}\text{ of }\frac{5}{\cancel{16}_{\,8}}\right)+\left(5\frac{8}{11}\div\frac{9}{22}\right)$$
$$=\tfrac{15}{56}+(\tfrac{63}{11}\div\tfrac{9}{22})$$
$$=\frac{15}{56}+\left(\frac{\cancel{63}^{\,7}}{\cancel{11}}\times\frac{\cancel{22}^{\,2}}{\cancel{9}}\right)$$
$$=\tfrac{15}{56}+14$$
$$=14\tfrac{15}{56}$$

Exercise 21

Simplify the following :—

1. $4\frac{1}{8}\times\frac{7}{33}\times\frac{6}{19}\div\frac{18}{19}$.

2. $3\frac{1}{3}$ of $5\frac{2}{5}\div3\frac{9}{10}$.

3. $4\frac{1}{5}+3\frac{3}{10}\div\frac{7}{10}$.

4. $93\frac{3}{11}-7\frac{5}{33}\div\frac{3}{10}$ of $\frac{5}{16}$.

5. $5\frac{1}{4}\div\frac{6}{16}+\frac{3}{4}$.

6. $7\frac{1}{3}\div1\frac{2}{9}-\frac{5}{6}$.

7. $\frac{4}{11}-\frac{1}{3}$ of $1\frac{2}{11}\div16\frac{1}{4}+2\frac{1}{2}$.

8. $55\frac{5}{11}-6\frac{1}{9}\times4\frac{7}{33}$.

9. $\frac{3}{11}\div\frac{6}{7}-\frac{3}{11}$.

10. $4\frac{1}{8}-3\frac{3}{4}$ of $\frac{16}{40}\div\frac{8}{15}$.

11. Write a rule for dividing by a Vulgar fraction.

Note the following—

(i) $3\frac{1}{2} \div \frac{4}{21} \times \frac{5}{7}$
$= \frac{7}{2} \times \frac{21}{4} \times \frac{5}{7} = 13\frac{1}{8}$.

In this case the sign $\div$ affects $\frac{4}{21}$ only.

(ii) $3\frac{1}{2} \div \frac{4}{21}$ of $\frac{5}{7}$
$= \frac{7}{2} \div (\frac{4}{21} \times \frac{5}{7})$
$= \frac{7}{2} \div \frac{20}{147}$
$= \frac{7}{2} \times \frac{147}{20} = 25\frac{29}{40}$.

In this case $\frac{4}{21}$ of $\frac{5}{7}$ must be regarded as one quantity.

12. Simplify: (i) $9\frac{1}{3} + \frac{5}{11}$ of $30\frac{1}{4}$; (ii) ($\frac{3}{8}$ of $\frac{7}{9}$)+($\frac{5}{11}$ of $35\frac{3}{4}$); (iii) $\dfrac{\frac{5}{8}+\frac{3}{7}}{\frac{3}{4}-\frac{1}{9}}$; (iv) $\dfrac{\frac{7}{16} \text{ of } \frac{9}{20}}{\frac{5}{11} \text{ of } 4\frac{5}{7}}$.

Fractional Values

Examples.—(i) *Find the value of* $3\frac{7}{9}$ of £16, 15s. $9\frac{3}{4}$d.
(ii) *Find the value in yards of* $\frac{3}{7}$ *of* $\frac{35}{99}$ *of* 1 *mile.*

(i) $3\frac{7}{9}$ of £16, 15s. $9\frac{3}{4}$d. $=(3 \times$ £16, 15s. $9\frac{3}{4}$d.$)+(\frac{7}{9}$ of £16, 15s. $9\frac{3}{4}$d.);
$=$ £50, 7s. $5\frac{1}{4}$d. $+ 7 \times$ £1, 17s. $3\frac{3}{4}$d.
$=$ £50, 7s. $5\frac{1}{4}$d. $+$ £13, 1s. $2\frac{1}{4}$d.
$=$ £63, 8s. $7\frac{1}{2}$d.

(ii) $\frac{3}{7}$ of $\frac{35}{99}$ of 1 mile $= \frac{\cancel{3}}{\cancel{7}}$ of $\dfrac{\overset{5}{\cancel{35}}}{\cancel{99}\ \underset{3}{\cancel{33}}}$ of $\dfrac{\overset{160}{\cancel{1760}}}{1}$ yds.
$= \frac{800}{3}$ yds.
$= 266\frac{2}{3}$ yds.

Exercise 22

Find the value of

1. $\frac{11}{12}$ of £13, 11s. 9d.

2. $\frac{5}{16}$ ton $+\frac{6}{7}$ cwt. $-39\frac{1}{2}$ lbs. $-1\frac{1}{7}$ cwt. (Answer in cwts. and lbs.).

3. $\frac{3}{7}$ of $\frac{5}{16}$ of $\frac{14}{15}$ of half a sovereign. (Answer in s. and d.)

4. $5\frac{1}{3}$ mins. $+2\frac{2}{3}$ hrs. $+5\frac{1}{2}$ secs. $+3\frac{1}{3}$ hrs. (Answer in hrs., etc.)

5. $1\frac{5}{11}$ furlongs $+3\frac{1}{8}$ miles -500 yds. (Answer in miles and yards.)

6. $7\frac{1}{4}$ galls. $+3\frac{1}{4}$ pts. $-5\frac{1}{4}$ qts. (Answer in gallons, etc.)

7. $\frac{15}{28}$ cwt. $+\frac{3}{4}$ ton -1600 lbs. (Answer in cwts. and lbs.)

8. $\frac{4}{7}$d. $+\frac{3}{5}$s. $+$ £$5\frac{1}{5}$.

9. $3\frac{1}{2}$ times £5, 11s. $3\frac{2}{3}$d.

10. Four times £2, 8s. $5\frac{1}{5}$d. $-\frac{2}{5}$ of £6, 11s. $3\frac{3}{5}$d.

Exercise 23

1. A boy was told to place a mark on a line $14\frac{1}{4}$ ins. long, at a point $4\frac{9}{16}$ ins. from one end. He measured from the wrong end. How far from the correct position did he mark the point ?

2. A man travelled $\frac{5}{16}$ of his journey by coach, $\frac{7}{20}$ by rail, and walked the last $6\frac{3}{4}$ miles. How far did he travel by rail ?

3. One man owns three-fourths of a ship, and another man the remainder. If the first man sells three-fourths of his share for £360, what is the value of the second man's share ?

4. Simplify

$$(a)\ \frac{\frac{1}{9} \text{ of } 1\frac{1}{4}\times 4\frac{1}{2}}{\frac{5}{56}\times 1\frac{1}{3}\times 3\frac{1}{2}};\quad (b)\ 2\frac{1}{4}\times 1\frac{1}{2}\div 1\frac{1}{8} \text{ of } 2\frac{2}{5}.$$

5. (*a*) Find the difference between two fractions, one of which is $\frac{3}{4}$, if their product is $\frac{2}{5}$.

(*b*) Simplify $\dfrac{4\frac{7}{8}-3\frac{5}{24}-\frac{1}{16}}{12\frac{1}{4}\div 4\frac{1}{5}}$.

6. How can you tell without actually working the sum that the following fractions added together are greater than one ?

$$\frac{5}{16},\ \frac{7}{20},\ \frac{7}{24},\ \frac{9}{32}.$$

7. Divide the greatest of the following fractions by the least :

$$\frac{7}{15},\ \frac{8}{9},\ \frac{5}{6},\ \frac{3}{10}.$$

8. How much is the fraction $\frac{7}{12}$ increased or diminished by adding 4 to both numerator and denominator ? How would the value of the fraction be altered if both numerator and denominator were multiplied by 4 ?

9. A boy spends $\frac{3}{5}$ of his money and then $\frac{1}{4}$ of the remainder. He has 4s. 6d. left. How much had he at first ?

10. Find the value of :

$$(a)\ 5\frac{1}{2}\times 4\frac{1}{2}+\frac{3}{5} \text{ of } 7\frac{1}{2}+8\frac{3}{10};\quad (b)\ 1\frac{29}{45}\div\left(\frac{2}{3}-\frac{4}{15}\right).$$

SECTION II

EXTENSION OF ARITHMETICAL PROCESSES

Decimal Fractions

When a fraction has a denominator 10 or a multiple of 10, such as 100, 1000, etc., it is spoken of as a Decimal Fraction.

Thus $\frac{7}{10}$ and $\cdot 7$ represent the same decimal fraction, and we shall speak of $\frac{7}{10}$ as the vulgar form and $\cdot 7$ as the decimal form.

When we wish to multiply a decimal by 10 we need only move the decimal point one place to the right. When multiplying a decimal by 100 we move the decimal point two places to the right, and so on, *e.g.*

$$78{\cdot}6 \times 10 = 786\,; \quad 78{\cdot}6 \times 100 = 7860.$$

When we wish to divide a decimal by 10 we move the decimal point one place to the left. When dividing a decimal by 100 we move the decimal point two places to the left, and so on, *e.g.*

$$78{\cdot}6 \div 10 = 7{\cdot}86\,; \quad 78{\cdot}6 \div 100 = {\cdot}786.$$

This shows how the Metric System provides for the easy changing of its weights and measures from one denomination to another.

EXERCISE 24

1. Express the following in vulgar form :—

(*a*) ·3, ·07, 5·157, 3·019, 1·0001, 4·003.
(*b*) ·0177, 2·8901, 7·07, 9·019, ·000011, 7·13.

2. Express the following in decimal form :—

(*a*) $\frac{3}{10}$, $\frac{7}{100}$, $4\frac{57}{1000}$, $3\frac{1}{2}$, $5\frac{1}{4}$, $2\frac{3}{4}$.
(*b*) $7\frac{391}{1000}$, $5\frac{1}{1000}$, $\frac{39}{10000}$, one millionth, seven thousandths, $\frac{73}{10000}$.

3. Give answers to the following in decimal form :—

(*a*) $\frac{3}{10}+\frac{7}{100}$, $\frac{99}{100}-\frac{17}{100}$, $\frac{1}{2}-\frac{3}{10}$, $\frac{1}{4}+\frac{37}{100}$, $\frac{1}{10}-\frac{1}{100}$, $\frac{3}{4}-\frac{1}{2}$.
(*b*) Three and a quarter+five and a half+half of a hundredth.

(*c*) $\frac{3}{10}+\frac{9}{100}+\frac{7}{1000}+\frac{73}{10}$.
(*d*) $\frac{1}{8}+\frac{1}{4}+\frac{1}{100}$.

4. A line 6 ins. in length is divided into 10 equal parts. Find the length of 4 of these equal parts.

5. A foot rule is divided into 10 equal parts. Find the length of (*a*) 3 of these equal parts, (*b*) 5 of them, (*c*) 2 of them.

6. Consider the number 3·14159. (*a*) Find the difference in value between the two 1's given. (*b*) Write the values of the 4 and the 9 in decimal form, and also in fractional form.

7. What numbers must 734 be divided by to give a dividend of (*a*) ·734, (*b*) 7·34, (*c*) 73·4, (*d*) ·00734 ?

8. What numbers must ·83 be multiplied by to give a product of (*a*) 83, (*b*) 830, (*c*) 8·3, (*d*) ·83 ?

9. If my walking stick is a yard long, what length remains after I have worn away ·1 of it ? What fraction of the original length remains ?

10. Write down the values of : $7{\cdot}35\times10$, $\cdot08\times100$, $\cdot85\div10$, $72{\cdot}98\div100$.

Addition and Subtraction

All operations in decimals may be performed in exactly the same manner as those in whole numbers—but special attention must be given to the position of the decimal point.

In addition and subtraction of decimals we must be careful to keep the decimal points in a vertical line. This will ensure that the tens, hundreds, etc., are in their correct positions.

Example (i).—*Add* 5·084, 64·03, ·0005, 720, 55·3, 7·08463.

(ii) *Find the difference between* 71·6 *and* 5·80001.

Working (i)		(ii)	
	5·084		71·6
	64·03		5·80001
	·0005		65·79999
	720		
	55·3		
	7·08463		
	851·49913		

EXERCISE 25

1. Add 384·2, 76·013, 458·901, ·0246, ·08, 56.

2. Find the sum of 42·6, 3·051, ·008, ·9034, 785·4.

3. (*a*) Find the sum of : nine-tenths of an inch, half of

one-twentieth of an inch, seven-hundredths of an inch, half of one-tenth of an inch, half of one-hundredth of an inch, four and seven-hundredths of an inch.

(*b*) Write down a number which is ten times their sum and another which is a hundred times their sum.

4. Ten readings of a thermometer are : 55·2° F., 50·6° F., 60° F., 54·6° F., 55·5° F., 59·1° F., 57·6° F., 54° F., 58·2° F., 56·8° F. Find the average temperature recorded.

5. Find the difference between :

(*a*) 7·84 and 30·001, (*b*) 54·2 and 100·095.
(*c*) 70 and 55·58, (*d*) ·8423 and 8·423.

6. What length of tape is left after cutting from a 60-inch length first 3·84 ins., then 15·2 ins., and then 17·05 ins. ?

7. Find the value of 8·34—7·123+15·055—3·391.

8. A reel of thread contains 100 yds. After five lengths, each measuring 8·73 ins., are cut off, what length remains ?

9. Find the perimeter of

(*a*) A square plot having one side 354·6 ft.
(*b*) A rectangular garden, 137·8 ft. by 74·6 ft.

10. Find the value of x when

$$84{\cdot}5+37{\cdot}384+{\cdot}055+x=560.$$

Multiplication

Examples.—(i) *If one bag of flour weighs* 38·58 *lbs., how much would* 46 *similar bags weigh ?*

(ii) Find the value of

(*a*) 38·58 × 4·6 ; (*b*) 38·58 × ·46 ; (*c*) 38·58 × ·046.

(i)
```
 38·58 lbs.
    46
 ----------
 1543·2
  231·48
 ----------
 1774·68 lbs.
```

(ii) (*a*) As 4·6 is one-tenth of 46, the answer will be one-tenth of the previous answer. That is 177·468.

(*b*) As ·46 is one-hundredth of 46, the answer will be $\frac{1}{100}$ of the first answer. That is 17·7468.

(*c*) ·046 is $\frac{1}{1000}$ of 46. Therefore the answer is 1·77468.

Collecting these examples, we see that

38·56 × 46 = 1774·68
38·56 × 4·6 = 177·468
38·56 × ·46 = 17·468
38·56 × ·046 = 1·7468

The number of decimal places in the product is always equal to the sum of the number of decimal places in the quantities multiplied together.

This shows the rule we must follow in multiplication of decimal quantities.

Exercise 26

1. $8 \cdot 79 \times 5$.	**6.** $59 \cdot 05 \times \cdot 72$.
2. $\cdot 7862 \times 18$.	**7.** $4 \cdot 185 \times \cdot 073$.
3. $568 \cdot 3 \times 291$.	**8.** $8 \cdot 02 \times \cdot 0004$.
4. $5 \cdot 861 \times 3 \cdot 7$.	**9.** $\cdot 0009 \times \cdot 006$.
5. $18 \cdot 007 \times \cdot 8$.	**10.** $7 \cdot 008 \times 5 \cdot 089$.

11. Find the value of $5 \cdot 137 \times (\cdot 02)^2$.

12. If 1 knot $= 1 \cdot 152$ miles per hour, how many miles does a steamer travel in a day if its speed is 30 knots ?

13. 1 in. $= 2 \cdot 54$ cm. Find in cm. the length of 1 ft. ; and 1 yd.

14. A cubic foot of petrol weighs 47·48 lbs. Find the weight in lbs. of 26 cu. ft.

15. A dishonest merchant uses a yard measure which is only 35·87 ins. in length. What length does he gain when he has used this measure 30 times ?

16. If my pen measures 7·75 ins., write down the length of 10, 100, 1000 such pens when placed end to end.

17. When x is divided by 17 the answer is 5·082. If there is no remainder find the value of x.

18. Multiply the sum of ·08 and ·75 by the product of ·76 and 200.

Division

Example i.—*Find the area of each part when* 843·75 *acres are divided into* 9 *equal parts.*

$$9\overline{)843 \cdot 75} \text{ acres}$$
$$93 \cdot 75 \text{ acres.}$$

Notes

1. The difficulty in division of decimals is the fixing of the decimal point. When dividing by a whole number, as above, the decimal points must be in a vertical line, thus keeping hundreds, tens, units, tenths, and hundredths in their correct positions.

2. All exercises in division may be expressed as fractions, thus :

$$843 \cdot 75 \div 9 = \frac{843 \cdot 75}{9} = 93 \cdot 75.$$

3. Remember that the value of a fraction remains unaltered if the numerator and denominator are multiplied or divided by the same quantity.

Example ii.—*Divide* ·9724 *by* ·017.

Method i.—By Note 2 above we may write this problem thus:

$$\frac{\cdot 9724}{\cdot 017}$$

By Note 3 above we may multiply this fraction by $\frac{1000}{1000}$ and then it becomes $\frac{972\cdot 4}{17}$, and having thus brought the denominator to a whole number, we may proceed as in Example i above, thus:

$$\begin{array}{r}
57{\cdot}2 \\
17\overline{)972{\cdot}4} \\
\underline{85\phantom{{\cdot}4}} \\
122\phantom{{\cdot}} \\
\underline{119\phantom{{\cdot}}} \\
34 \\
\underline{34}
\end{array}$$

The divisor may always be brought to a whole number in this way, and this method may be followed. Remember to multiply or divide the dividend by the same quantity used to bring the divisor to a whole number.

Method ii.—By Note 3 above $\frac{\cdot 9724}{\cdot 017}$ may be written $\frac{97\cdot 24}{1\cdot 7}$

We have now expressed the divisor in such a way that it contains one whole number, and this enables us to fix the decimal point by inspection.

When using the second method we must arrange that the divisor contains one whole number, and then the position of the decimal in the quotient is readily fixed.

Remember, again, to multiply or divide the dividend by the same quantity used to bring the divisor to its new form.

There are other methods of working, but in the final result all may be proved by remembering that the sum of the decimal places in the divisor and quotient must always equal the number of decimal places in the dividend.

Compare this with the rule for the multiplication of decimals given on p. 22.

Example iii.—*Divide* ·8572 *by* ·019.

By Method i. $\quad \frac{\cdot 8572}{\cdot 019} = \frac{857\cdot 2}{19}$

By Method ii. $\quad \frac{\cdot 8572}{\cdot 019} = \frac{85\cdot 72}{1\cdot 9}$

Working

(i)	(ii)
19)857·2(45·1157 . . .	1·9)85·72(45·1157 . . . .
76	76
97	97
95	95
22	22
19	19
30	30
19	19
110	110
95	95
150	150
133	133
17	17

Note that the result does not work out exactly. In such a case we must give the answer as correctly as is required. Study these statements :

45·1	is the correct answer to one decimal place.
45·12	,, ,, ,, two decimal places.
45·116	,, ,, ,, three ,, ,,

When the figure obtained in the quotient in the next place beyond the number of decimals required is 5 or a number greater than 5, we increase the last figure in the number of figures required by 1. When the figure obtained in the quotient in the next place beyond the number of decimals required is less than 5, we make no change in the figures required.

Exercise 27

1. 782·03÷5.
2. 84·6532÷50.
3. 748·03÷5000.
4. ·843÷·3.
5. ·7845÷·015.
6. ·12348÷90.
7. 6828÷·003.
8. ·328÷72.
9. ·0381÷·0003.
10. ·087÷·3.

Work Exercises 11 to 20 correct to 3 places of decimals (if necessary).

11. 3·884÷·052.
12. 45·72÷·0007.
13. ·012÷·95.
14. 5÷3·8.
15. 8·1÷55·2.
16. ·7351÷73·5.
17. ·332÷9·09.
18. 84·3÷·015.
19. ·03÷7·7.
20. 9·8÷18·011.

To change a Vulgar Fraction to a Decimal Fraction we must divide the numerator by the denominator, and proceed exactly as we do in the division of decimals.

Example.—Change $\frac{5}{16}$ to a decimal.

```
  5
 ――
16)5·0(·3125
    48
    ―
     20
     16
     ―
      40
      32
      ―
       80
       80
       ―
```

If the decimal does not terminate, we can find the value to the number of decimals required as shown on p. 24.

Exercise 28

1. Change to decimal fractions:

$\frac{1}{8}$, $\frac{1}{4}$, $\frac{3}{8}$, $\frac{1}{2}$, $\frac{5}{8}$, $\frac{3}{4}$, $\frac{7}{8}$, and

memorise the results.

2. Express as decimals: $\frac{3}{50}$, $\frac{8}{25}$, $\frac{7}{32}$, $\frac{5}{32}$, $\frac{6}{25}$

3. Express as decimals: $\frac{7}{125}$, $\frac{93}{64}$, $\frac{55}{16}$, $\frac{300}{96}$, $\frac{33}{64}$

4. Express as decimals correct to three decimal places:

$\frac{5}{6}$, $\frac{3}{7}$, $\frac{19}{33}$, $\frac{15}{18}$, $\frac{77}{12}$

5. Express as decimals correct to the hundredths figure:

$\frac{7}{11}$, $\frac{215}{3}$, $\frac{71}{18}$, $\frac{44}{9}$, $\frac{15}{7}$

6. Express as decimals correct to the thousandths figure:

$\frac{2}{3}$, $\frac{7}{9}$, $\frac{3}{11}$, $\frac{1}{7}$, $\frac{2}{9}$

7. Write a rule by which we may tell by inspection whether a fraction can be expressed as an exact decimal.

8. Express as a decimal: $\frac{4}{1000}$, $\frac{50}{1000}$, and $\frac{800}{1000}$, and thence obtain the value of $\frac{854}{1000}$ as a decimal.

9. Express as a decimal: $\frac{4}{8}+\frac{3}{5}+\frac{17}{100}+\frac{4}{32}$. Work by the shortest method.

10. Convert each fraction to a decimal form and then express decimally: $(\frac{3}{8}\times 10\times 100)-(\frac{1}{2}+\frac{3}{4}+\frac{7}{8})$.

Exercise 29

1. Find the value of $\dfrac{3{\cdot}25+{\cdot}85}{3{\cdot}75\div1{\cdot}5}$

2. Steel expands $\frac{1}{150,000}$ for each 1° F. rise in temperature. What will be the increase in the length of a steel rod 40 ft. long for 315° F. rise of temperature? Give the answer in inches correct to the third significant figure.

3. What is the cost in £, s. d. of 425 ozs. of silver at 32·125 pence an ounce?

4. Simplify, expressing the result as a decimal, correct to two places, $(\frac{15}{32}+\frac{8}{9}-\frac{5}{16})\times{\cdot}35$.

5. Express in tons, cwts., qrs., and lbs., the sum of 1·35 tons, 17·45 cwts., 3·25 qrs., 17·875 lbs.

6. If 2 tons 5 cwts. 2 qrs. of coal cost £9·1, what is the cost in shillings of 1 cwt.?

7. Divide 36 by $\frac{3}{7}$. Explain why the answer is greater than the number divided.

8. Simplify $\dfrac{4-1\frac{1}{3}+\frac{1}{4}-\frac{1}{2}}{6+1\frac{3}{4}-\frac{1}{4}+\frac{1}{2}}\div\frac{3}{11}$

9. Divide 30s. among X, Y, and Z, so that X receives twice as much as Y and Y three times as much as Z.

10. Find the value of $\dfrac{4{\cdot}25}{8{\cdot}1}\times\dfrac{2{\cdot}7}{8{\cdot}5}$

11. Find the cost of 99 yds. of silk at 4s. $11\frac{2}{3}$d. per yd.

12. Find the eleventh part of £15, 5s. $2\frac{3}{5}$d. exactly.

13. A line AB is $5\frac{1}{2}$ ins. in length. It is divided at the point C into two parts, so that AC$=2\frac{1}{2}$ times CB. Give the lengths of AC and CB.

14. If a line $5\frac{19}{20}$ ins. in length is divided into 50 equal parts, what length will 17 of these parts be?

15. Insert a fraction between $\frac{49}{72}$ and $\frac{39}{64}$ so that its difference from each is exactly the same.

16. Simplify $\dfrac{63-0{\cdot}63+0{\cdot}255}{4{\cdot}72-3{\cdot}05}$

17. How many times can a jug which holds 0·625 of a pint be filled from a cask which contains 6 galls. 1 qt. 1 pt., and what part of a pint will be left in the cask?

18. How much is a draper's yard-stick too short if a buyer of $17\frac{1}{2}$ yds. of calico receives 4·375 ins. too little?

19. A man walks at $3\frac{3}{4}$ miles per hour, taking steps of length

33 ins. Find (i) how many feet he walks per second; (ii) how many steps he takes per minute.

20. Divide £7, 10s. $7\frac{3}{4}$d. between A and B so that B's share is worth ·4 of A's share.

Unitary Method

Example.—If 10 *horses cost* £425, *what would be the cost of* 25 *horses?*

We can find the price of 1 horse by dividing £425 by 10, and then we can find the price of 25 horses by multiplying the price of 1 by 25. We may state it thus:

If 10 horses cost £425

$$\text{Then } 1 \text{ horse costs } \frac{£425}{10}$$

$$\therefore 25 \text{ horses cost } \frac{£425 \times \overset{5}{\cancel{25}}}{\underset{2}{\cancel{10}}}$$

$$= \frac{£2125}{2} = £1062, 10\text{s. } 0\text{d.}$$

or, more briefly . . . If 10 horses cost £425.

[Now do the next step mentally: 1 horse costs $\frac{£425}{10}$, and then write at once the third line.]

$$\text{Then } 25 \text{ horses cost } \frac{£425}{10} \times 25.$$

Or, we may reason that 25 horses will cost $\frac{25}{10}$ of £425. When two quantities are so related that an *increase* in one causes a corresponding *increase* in the other, or *vice versa*, the quantities are said to be in *Direct Proportion.*

When, however, two quantities are so related that an *increase* in one produces a corresponding *decrease* in the other, or *vice versa*, the quantities are said to vary indirectly or to be in *Inverse Proportion.*

Example.—A garrison of 2000 *besieged men were provided with food for* 20 *days. If there had been* 800 *men, how long would the food have lasted, reckoning at the same rate?*

If the food would last 2000 men for 20 days

Then at this rate it would last 1 man for 20×2000 days

$$\therefore \text{At this rate it would last } 800 \text{ men for } \frac{\cancel{20} \times \overset{25}{\cancel{2000}}}{\cancel{800}} = 50 \text{ days.}$$

[Note the second line of the above statement. The *decrease* in men produces a corresponding *increase* in the days the food will last.]

Exercise 30

1. A man earns £4, 10s. 0d. a week when working 54 hrs. weekly. If he continues to be paid at this rate, how much will he receive for a working week of 48 hrs.?

2. Find the cost of 958 hats at 77s. per dozen.

3. When 3 tons 15 cwts. are carried 50 miles for a certain charge, how far should 10 tons 5 cwts. be carried for the same charge?

4. If 17 trucks of equal weight are found to weigh 76 tons 10 cwts., what will 7 of the trucks weigh?

5. When 3 tons 15 cwts. 3 qrs. of waste metal are bought for £150, 10s. 0d., find the cost of 12 cwts. $2\frac{1}{2}$ qrs. at the same rate.

6. If $97\frac{1}{2}$ yds. of silk cost £30, 10s. 0d., find the cost of 65 yds. at the same rate.

7. If 18 men can build a wall in 15 days, how long would it take 30 men if the same rate per man were maintained?

8. A tap can fill a cistern in 20 minutes; another tap can fill it in 16 minutes; how long would it take to fill the cistern if both taps were running together?

9. A garrison of 1200 men was provisioned with food for 24 days, but 600 men joined the garrison at once. How long would the food last then?

10. When $\frac{1}{2}$ of $\frac{9}{10}$ of a ship is worth £5000, what is the value of $\frac{9}{10}$ of $\frac{1}{10}$ of it?

11. If a man pays a tax of 3s. 6d. in the pound, what is his original income when his net income after he has paid his tax is £396?

12. Find the cost of 1000 eggs at 9d. a dozen.

13. If a train running at $47\frac{1}{2}$ miles an hour takes 1 hr. 36 mins. on a journey, how long will it take if its speed was increased by $3\frac{1}{8}$ miles an hour?

14. Two persons start at the same time walking towards each other from places 3 miles apart. If one of them is walking at 4 miles an hour and they meet 24 minutes after their start, how fast is the other walking?

15. A knot is a speed of 6080 ft. per hour. What distance will an aeroplane, flying at 60 miles an hour, gain in 20 mins. on a destroyer steaming at 30 knots?

(Give the answer in miles and yards to the nearest yard.)

16. If 3 cwts. of sugar cost £3, 10s. 0d., how many cwts. can be bought for £26, 5s. 0d.?

17. If 12 horses or 20 cattle eat the hay in a barn in 16 days, in what time will 20 horses and 24 cattle eat it ?

18. Two taps together will fill a tank in $1\frac{1}{2}$ hrs. ; one of them alone will fill it in $2\frac{1}{2}$ hrs. How long would it take the other tap running alone ?

19. A, B, and C can do a piece of work in 5 days ; A and B can do it in 8 days ; in what time can C alone do it ?

20. How many men will perform a piece of work in $16\frac{7}{8}$ days if it takes 20 men 27 days ?

Ratio

Ratio is the relation which one quantity bears to another with respect to magnitude. We make the comparison by determining what multiple, or what part, the first quantity is of the second.

When comparing two *concrete* quantities, these quantities must be of the same kind. We may compare length with length, area with area, a number of books with another number of books, but not a number of men with a number of horses—the quantities must be of the same kind.

Ratio is expressed by placing two dots between the two numbers, or the two quantities compared. Thus the ratio of 8 to 2 is shown by 8 : 2. The first number or quantity is spoken of as the antecedent, and the second number or quantity as the consequent.

The ratio 8 : 2 may also be expressed thus : $\frac{8}{2}$. The ratio of 25 books to 10 books may be expressed as $\frac{25}{10}=\frac{5}{2}$. The ratio of £3, 2s. 6d. to £5, 12s. 6d. $=\frac{25}{45}=\frac{5}{9}$. It is generally advisable to express the fraction in its lowest terms.

Note that the quantities may be concrete, but the ratio is abstract. Thus, the ratio that 2s. 6d. bears to 10s. is 1 : 4 or $\frac{1}{4}$. 2s. 6d. and 10s. are concrete, but the fraction $\frac{1}{4}$ is abstract.

Example.—What fraction of £5, 11*s.* 6*d. is* £1, 12*s.* $6\frac{1}{4}$*d. ?*

$$\text{Fraction}=\frac{\text{£1, 12s. }6\frac{1}{4}\text{d.}}{\text{£5, 11s. 6d.}}=\frac{1561}{5352}=\frac{7}{24}.$$

Exercise 31

What fraction of

1. 19s. $1\frac{1}{4}$d. is 8s. $2\frac{1}{4}$d. ?

2. £6, 0s. 9d. is £2, 13s. 8d. ?

3. £14, 1s. 6d. is £5, 17s. $3\frac{1}{2}$d. ?

4. 13 cwts. 3 qrs. 21 lbs. is 9 cwts. 3 qrs. 23 lbs. ?
5. 1 ton is 2 cwts. 96 lbs. ?
6. $2\frac{1}{2}$ miles is 264 yds. ?
7. 33 yds. is 5 ft. 6 ins. ?
8. 4 galls. 3 qts. is $4\frac{3}{4}$ pts. ?
9. £2 is £5, 15s. 6d. ?
10. 363 sq. yds. is 1 acre ?

Exercise 32

1. Find the following ratios :—

(*a*) £3, 7s. 6d. to £4.
(*b*) $2\frac{1}{2}$ qts. to 3 galls.
(*c*) $1\frac{1}{2}$ miles to 7 furlongs.
(*d*) $\frac{1}{2}$ hr. to 2 days.

2. What is meant by a notice placed by the side of a railway track which says, "Gradient 1 in 1500" ?

3. Express the following ratios in decimals :—

(*a*) Population of 150,000 to population of 7,500,000.
(*b*) Speed of sound 1120 ft. per sec. to speed of light 186,000 miles per sec.
(*c*) Height of room 12 ft. 6 ins. to length of room 31 ft. 3 ins.

4. Express as ratios the following scales :—

(*a*) 1 in. to 1 ft.; 1 cm. to 1 m.
(*b*) 1 in. to 1 yd.; ·75 cm. to 2 cm.
(*c*) 1 in. to 1 chain; 15 mm. to 15 m.
(*d*) 1 in. to 1 mile; 1 dm. to 2 m.

5. The area of the Isle of Man is 230 sq. miles. The area of a map of the Isle is 27 sq. ins. Find the ratio $\frac{\text{Area of Map}}{\text{Area of Island}}$.

6. 273 c.c. of gas at 0° C. become 274 c.c. at 1° C. What ratio does the expansion bear to the original volume ?

7. The Norman kings reigned from 1066 A.D. to 1154 A.D. What ratio does the length of the reign of William I. (1066 A.D. to 1087 A.D.) bear to the time of the Norman Dynasty ?

8. A parcel valued at £8 is insured for 6d. What is the ratio of the insurance fee to the value of the goods ?

9. The area of a plan is 28 sq. ins. The area of the figure

it represents is 1 sq. ft. 66 sq. ins. Find the ratio of the area of figure to the area of plan.

10. A piece of indiarubber is 1 ft. 6 ins. long. It is stretched to 2 ft. 3 ins. Find ratio of the new length to the original length.

11. In fig. 3 measure the lines AB, CD, EF, OB, OD, OF, and find the ratios $\frac{AB}{OB}$, $\frac{CD}{OD}$, $\frac{EF}{OF}$.

12. If a pole 18 ft. high casts a shadow $24\frac{1}{2}$ yds. long, find the ratio of the length of the shadow to the length of the pole. Draw a sketch.

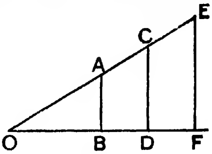

FIG. 3.

Decimalisation

Example.—Express 3 yds. 2 ft. 5 ins. as a decimal of 50 yds. 1 ft. 6 ins., correct to the second decimal place.

$$\text{Fraction} = \frac{3 \text{ yds. } 2 \text{ ft. } 5 \text{ ins.}}{50 \text{ yds. } 1 \text{ ft. } 6 \text{ ins.}} = \frac{137}{1818}.$$

Decimal = ·08.

EXERCISE 33

Express the second of the following quantities as a decimal of the first, as above:—

1. 11s. $0\frac{1}{2}$d.; 6s. $7\frac{1}{2}$d.
2. £2, 7s. 11d.; £2, 3s. $1\frac{1}{2}$d.
3. £3; £1, 17s. 6d.
4. 1 day 18 hrs. 40 mins.; 16 hrs.
5. A leap year; $91\frac{1}{2}$ days.
6. 4 kg.; 3 gms.
7. 3 tons; 90 stone.
8. 3 galls. 1 qt.; 13 pts.
9. 2 litres; 5 centilitres.
10. 121 sq. yds.; $\frac{1}{2}$ acre.
11. £3, 10s. 0d.; 5s. $5\frac{1}{2}$d. } (correct to 3rd decimal place).
12. $3\frac{1}{2}$ miles; 1 furlong 1 yd. }

Consider the following:—

2s. = $\frac{1}{10}$ of £1 = £·1.

Hence	4s. = £·2.	12s. = £·6.
	6s. = £·3.	14s. = £·7.
	8s. = £·4.	16s. = £·8.
	10s. = £·5.	18s. = £·9.

Again $\quad$ 1s. $= \frac{1}{20}$ of £1 = £·05.

Hence we may decimalise any number of shillings by multiplying the number by £·05.

Again, $\quad$ 6d. $= \frac{1}{40}$ of £ = £·025.

Hence $\quad$ 1s. 6d. = £·05 + £·025 = £·075.

And $\quad$ 5s. 6d. = £·25 + £·025 = £·275.

A simple rule underlies all the previous working, namely :—

To decimalise shillings and sixpences :

(i) Multiply the shillings by 5, and the result is hundredths of £1.

(ii) Add £·025, which is the value of the sixpence.

Example.—Decimalise 10*s.* 6*d. and* 17*s.* 6*d.*

(*a*) By rule $\frac{£\ 10 \times 5}{100} + £{\cdot}025 = £{\cdot}5 + £{\cdot}025 = £{\cdot}525.$

(Do not cancel.)

(*b*) By rule $\frac{£\ 17 \times 5}{100} + £{\cdot}025 = £{\cdot}85 + £{\cdot}025 = £{\cdot}875.$

Exercise 34

Express as decimals of £1 :—

(*a*) 13s., 5s., 19s., 17s., 15s., 11s., 9s., 7s., 3s.

(*b*) 3s. 6d., 4s. 6d., 6s. 6d., 19s. 6d., 10s. 6d., 18s. 6d., 17s. 6d., 15s. 6d., 14s. 6d., 13s. 6d., 12s. 6d., 11s. 6d., 9s. 6d., 8s. 6d., 7s. 6d., 5s. 6d., 2s. 6d.

De-decimalisation

Example (i).—*Express* £7·5625 *in £, s. d.*

Example (ii).—*Express* 5·735 *tons in tons, cwts., and lbs.*

$$\left.\begin{array}{rr} & £ \\ & 7{\cdot}5625 \\ & \underline{20} \\ \text{s.} & 11{\cdot}25\cancel{00} \\ & \underline{12} \\ \text{d.} & 3{\cdot}\cancel{00} \end{array}\right\} = £7,\ 11\text{s. } 3\text{d.}$$

$$\left.\begin{array}{rr} & \text{tons} \\ & 5{\cdot}735 \\ & \underline{20} \\ \text{cwts.} & 14{\cdot}7\cancel{00} \\ & \underline{112} \\ \text{lbs.} & 78{\cdot}4 \end{array}\right\} = 5 \text{ tons } 14 \text{ cwts. } 78{\cdot}4 \text{ lbs}$$

Exercise 35

1. Express in £, s. d. :—

£3·253125 ;	£·009375 ;	£4·028125 ;
£7·815625 ;	£·75625 ;	£1·055625 ;
£4·071875 ;	£·421875 ;	£8·8375 ;
£·671875 ;	£·634375 ;	£12·928125.

2. Express in tons, cwts., lbs. (correct to the nearest lb.) :—

3·625 tons ;	·8725 tons ;	18·1725 tons ;
12·055 tons ;	·6515 tons ;	6·4375 tons ;
18·605 tons ;	·375 tons ;	8·1762 tons.

3. Express in the usual form :—

£14·421875 ;	12·875 cwts.
6·325 sq. miles ;	4·823 galls.
4·276 cu. ft. ;	6·3125 shillings.

[*Work the following by decimalisation.*]

4. A man spends £6, 7s. $1\frac{1}{2}$d. out of his weekly wage of £7. What decimal fraction of his wage does he save ?

5. The rent of a house is ·078125 of its value. If the value is £800, what is the rent in £, s. d. ?

6. Find by inspection the value of 6·725 tons.

7. How many tickets at 3s. 9d. each must be sold to bring in £37, 10s. ?

8. Find the cost of 100 desks at £1, 5s. 9d. each.

9. A grocer serves 2000 customers with $3\frac{1}{2}$ lbs. of flour each. How many tons of flour has he served ?

10. If 10 million tax-payers together contribute £4,553,125, what is the average contribution of each in £, s. d. ?

11. Find in gallons the total capacity of a tank which is filled by 100 buckets of water each holding 2 galls. 1 qt. 1 pt.

12. Find the cost of 500 bales of cotton at 14 guineas per bale.

13. Find the cost of 100,000 bricks at 35s. 6d. per hundred.

14. Find the cost of 750 shares at £3, 2s. 6d. each.

15. Find the cost of printing one million tickets at £1, 7s. 6d. per thousand.

16. A merchant bought 1750 tons of coal at £1, 15s. 0d. per ton and sold at £2, 2s. 6d. per ton. Find his profit.

Percentage

Per cent. = per centum = per hundred.
3 per cent. = 3 per hundred.
£4 per cent. = £4 per £100.
Per cent. is sometimes written p.c. or %.
Thus 5% = five per cent. = five per hundred.

$6\% = 6 \text{ per } 100 = \frac{6}{100}$.

Thus $6\% \text{ of } 5000 = \frac{6}{100} \text{ of } 5000 = 300$.

$4\% \text{ of } £200 = \frac{4}{100} \text{ of } £200 = £8$.

Exercise 36

1. What fractions represent 2 per cent., $2\frac{1}{2}$ per cent., 9 per cent., $9\frac{1}{2}$ per cent., $33\frac{1}{3}$ per cent., 80 per cent., 200 per cent. ?

2. Find the value of 5 per cent. of 85, 7 per cent. of 70, $12\frac{1}{2}$ per cent. of £50, 4 per cent. of 25s., $3\frac{1}{2}$ per cent. of 16s. 8d.

3. What fractions represent 50 per cent. of £1, 25 per cent. of £1, 20 per cent. of £1, 15 per cent. of £1, 10 per cent. of £1, $7\frac{1}{2}$ per cent. of £1, 5 per cent. of £1, $3\frac{3}{4}$ per cent. of £1, $2\frac{1}{2}$ per cent. of £1, $1\frac{1}{4}$ per cent. of £1 ?

4. What percentages are equivalent to the following fractions : $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{3}$, $\frac{3}{4}$, $\frac{1}{8}$, $\frac{1}{6}$, $\frac{3}{1}$, $\frac{9}{2}$?

5. What per cent. are : £5 of £30, £2 of £10, 3 cwts. of 20 cwts., 14 lbs. of a cwt., 1 ft. of 1 yd., 30 boys of 45 boys, $\frac{1}{2}$d. of 6d. ?

Profit and Loss

The price at which a man buys goods is known as the *Cost Price* (C.P.), and the price at which he sells them is spoken of as the *Selling Price* (S.P.). Thus, if he buys goods for £100 and sells them for £110, his gross profit is £10.

It should be noted that whenever an article is sold at a certain price, that price is regarded as the C.P. by the buyer and as the S.P. by the seller.

The *percentage* profit (or loss) may be reckoned in many ways. Thus a merchant may find his percentage profit on (*a*) the price the goods cost him, (*b*) the price at which he sells his goods, (*c*) the money he has invested in his business, and so on. We shall indicate clearly in this book on what money the pupil is expected to find the percentage profit.

Example.—What is the S.P. of a wireless apparatus which cost £15, and was sold at a gain of 5 per cent. on the C.P. ?

Method (i). S.P. = £15 + $\frac{1}{20}$ of £15 = £15, 15s. 0d.

Method (ii). When C.P. is £100 then S.P. = £105

„ „ £15 „ = £1·05 × 15

= £15, 15s. 0d.

Exercise 37

1. A bill amounted to £50, 10s. 0d. If $2\frac{1}{2}$ per cent. were deducted, find the net amount paid.

2. Mr Brown bought 75 lbs. of copper which was invoiced at £170, 10s. 0d. per ton. Before paying the bill $12\frac{1}{2}$ per cent. was deducted from the gross amount. Find how much money he actually paid.

3. A grocer buys sugar at 46s. 8d. a cwt. At what price per lb. must he sell it so as to gain 10 per cent. on his outlay ?

4. At what price must an article which costs 18s. 6d. be sold to produce a profit of 15 per cent. on the outlay ?

5. In weighing butter a grocer wastes $1\frac{1}{2}$ per cent. How much does he waste in weighing 5 cwts. 4 lbs. ?

6. 56 lbs. of tea are bought for 2s. per lb. One stone is sold at 2s. per lb. and the rest at a gain of $2\frac{1}{2}$ per cent. on the C.P. How much profit is made ?

7. A cow cost a farmer £28, 10s. 0d., and he sold her at a reduction of $2\frac{1}{2}$ per cent. What was the selling price ?

8. Two grocers, A and B, each buy 2 cwts. of butter at £11, 8s. 0d. per cwt. A sells the whole of his purchase for £25, 5s. 0d. ; B sells his at a profit of $12\frac{1}{2}$ per cent. on the C.P. Which gained the more, and by how much ?

9. A pawnbroker lent Mr R. £4 on a watch worth £5, 10s. 0d. The watch was redeemed and the pawnbroker made a profit of 35 per cent. on the value of the watch. How much did Mr R. pay ?

Example.—A bicycle was bought for £10 *and sold for* £12, 10*s.* 0*d. Find the gain per cent. on the buying price.*

Method (i). The gain on £10 $=£2\frac{1}{2}$.

„ „ £100 $=£\dfrac{2\frac{1}{2}}{10}\times 100=£25.$

A gain of £25 on £100 $=\underline{25\%}$.

Method (ii). The gain on £10 $=£2\frac{1}{2}$.

Fractional gain $=\dfrac{2\frac{1}{2}}{10}=\dfrac{1}{4}$.

And $\frac{1}{4}=\frac{25}{100}=\underline{25\%}$.

Exercise 38

1. The following table shows the C.P. and the S.P. of certain articles. Find the gain per cent. in each case, (i) on the C.P., (ii) on the S.P.

	Cost Price.	Selling Price.
Piano	£30	£50
Bicycle	£8	£12
Carpet	£4, 10s. 0d.	£6, 15s. 0d.
Table	£3, 5s. 0d.	£7

2. Find the percentage gain on the C.P. of a fountain pen which was bought for 8s. and sold for 10s. 8d.

3. If an article which cost 9d. is sold for $10\frac{1}{2}$d., what percentage profit is made on the outlay? How many such articles must be sold to realise a total profit of £1?

4. Butter was bought at £8, 10s. 0d. per cwt., and the trader's expenses were £1 per cwt. Find the gain per cent. by the trader on his outlay when he sells the butter at 2s. 6d. a lb.

5. Oranges are bought at the rate of 10 for 8d., and sold at the rate of 8 for 10d. What is the gain per cent. on the C.P.?

6. A grocer mixes 12 lbs. of tea at 2s. per lb. with 25 lbs. of tea at 2s. 3d. per lb. He sells the mixture at 2s. 9d. per lb. What is his gain per cent. on the C.P.?

7. What is the percentage loss to a manufacturer if a tradesman who owes him £500 can pay only $\frac{5}{8}$ of his debt?

8. A man builds 3 houses at a cost of £600 each. He sells one for £700, one for £632, one for £820; find his percentage profit on the C.P. (i) in each case, (ii) on the whole transaction.

9. The price of a certain kind of tobacco has risen from 9d. to 1s. 3d. per ounce, and a man has reduced his annual consumption of it by one-fourth. By how much per cent. has his annual expenditure on it increased?

10. In a certain half-year the gross receipts of a railway company were £3,084,956, and the working expenses were £2,846,305. Find correct to two decimal places what per cent. of the gross receipts the working expenses were.

Example.—By selling a house for £880 *the owner gained* 10 *per cent. on the cost price. Find the cost price.*

Method (i). This selling price represents the cost price + $\frac{1}{10}$ of the cost price, *i.e.* $\frac{11}{10}$ of the cost price.

$\therefore \frac{11}{10}$ of cost price = £880.

and $\frac{10}{10}$,, ,, $= £\frac{880 \times 10}{11} = £800.$

[Check.—$800 + \frac{1}{10}$ of $800 = £880$.]

Method (ii). Cost Price may be assumed to be £100. Then Selling Price = £110.

						C.P.
∴ When 10% is gained, S.P. of				£110	indicates	£100
And	,,	,,	,,	1	,,	$£\frac{100}{110}$
,,	,,	,,	,,	880	,,	$£\frac{100 \times 880}{110}$
						= £800.

3. A grocer buys sugar at 46s. 8d. a cwt. At what price per lb. must he sell it so as to gain 10 per cent. on his outlay ?

4. At what price must an article which costs 18s. 6d. be sold to produce a profit of 15 per cent. on the outlay ?

5. In weighing butter a grocer wastes $1\frac{1}{2}$ per cent. How much does he waste in weighing 5 cwts. 4 lbs. ?

6. 56 lbs. of tea are bought for 2s. per lb. One stone is sold at 2s. per lb. and the rest at a gain of $2\frac{1}{2}$ per cent. on the C.P. How much profit is made ?

7. A cow cost a farmer £28, 10s. 0d., and he sold her at a reduction of $2\frac{1}{2}$ per cent. What was the selling price ?

8. Two grocers, A and B, each buy 2 cwts. of butter at £11, 8s. 0d. per cwt. A sells the whole of his purchase for £25, 5s. 0d. ; B sells his at a profit of $12\frac{1}{2}$ per cent. on the C.P. Which gained the more, and by how much ?

9. A pawnbroker lent Mr R. £4 on a watch worth £5, 10s. 0d. The watch was redeemed and the pawnbroker made a profit of 35 per cent. on the value of the watch. How much did Mr R. pay ?

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" " $£100 = £\frac{2\frac{1}{2}}{10} \times 100 = £25.$

A gain of £25 on £100 = __25%.__

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Fractional gain $= \frac{2\frac{1}{2}}{10} = \frac{1}{4}.$

And $\frac{1}{4} = \frac{25}{100} =$ __25%.__

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Table	£3, 5s. 0d.	£7

A further reduction of 5 per cent. from the invoice price is made if cash is paid on delivery. What does the wheel cost then ?

8. A man whose salary was £570 was given a bonus of $33\frac{1}{3}$ per cent. on his salary, and £40 as well. By how much per cent. was his income increased ?

9. B is 25 per cent. greater than A. C is 25 per cent. greater than B. How much per cent. is C greater than A ?

10. A manufacturer sells an article to a wholesale dealer, who sells it to a retail dealer, who sells it to a customer for £7, 10s. 0d. The manufacturer, wholesale dealer, and retail dealer make profits of 50 per cent., $33\frac{1}{3}$ per cent., 25 per cent. respectively on the C.P. How much did it cost to manufacture the article ?

11. A merchant whose goods were marked to give 50 per cent. profit on his C.P. gave his customers 30 per cent. discount. What was his final profit on the C.P. ?

12. A man sold a piano at a profit of $33\frac{1}{3}$ per cent. on the C.P., and with the money he bought another which he sold for £36, and lost 25 per cent. on the C.P. Find the profit on the first one.

Exercise 40

1. A grocer bought 5 cwts. 2 qrs. 16 lbs. of cheese for £19, 15s. 0d. What would be his rate per cent. of profit on the outlay if he sold the cheese at $10\frac{1}{2}$d. per lb. ?

2. Three men own a business worth £10,020. One owns 15 per cent. of it, and the second 45 per cent. What is the value of the third owner's share ?

3. By selling an article for 55s. I gain $7\frac{1}{2}$ per cent. on the C.P. What should I gain or lose per cent. on the C.P. by selling it at 52s. 6d. ?

4. A man gained 15 per cent. on the C.P. by selling his house for £750. What percentage on the C.P. would he have gained by selling it for £850 ?

5. I bought a watch for £5, 5s. 0d., and sold it at a loss of $6\frac{2}{3}$ per cent. on what I paid. For how much did I sell the watch ?

6. My gross interest from an undertaking was £25, 10s. 0d. Find the net income from it after deducting : (i) 10 per cent., (ii) 5 per cent., (iii) $7\frac{1}{2}$ per cent., (iv) $2\frac{1}{2}$ per cent.

3. A grocer buys sugar at 46s. 8d. a cwt. At what price per lb. must he sell it so as to gain 10 per cent. on his outlay ?

4. At what price must an article which costs 18s. 6d. be sold to produce a profit of 15 per cent. on the outlay ?

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Interest

The following is a type of advertisement often seen in the daily and weekly newspapers :—

> The Corporation of —— are prepared to receive loans of £100 and upwards for a term of years at $4\frac{1}{2}$ per cent. per annum. Interest is payable half-yearly.

This means that the above Corporation wish to borrow money. For the use of this money they are prepared to pay £$4\frac{1}{2}$ every year for every £100 borrowed, and at the end of a certain time they will repay the original sum borrowed. This payment for the loan of money is known as Interest. The Interest paid depends upon the amount lent and the length of time for which it is lent.

Notes

1. The Principal is the money lent—*i.e.* the money owned by the lender.
2. Per cent.=per centum=per hundred.
3. Per annum=per year.
4. The Amount=the Principal+the Interest.
5. When the interest is paid at definite times, such as quarterly, half-yearly, yearly, it is spoken of as Simple Interest.
6. When the interest is not paid out to the lender but is added to the principal, and so forms a new principal for another definite length of time, it is spoken of as Compound Interest.
7. %=per cent; p.a.=per annum.

Example.—Find the total Simple Interest on £750 in 3 years at 4 per cent. per annum.

Interest on £100 in 3 yrs. at 4% p.a. $=£4 \times 3$.

„ „ £750 „ „ „ $=£\frac{750 \times 4 \times 3}{100}=£90$.

Exercise 41

Find the total simple interest paid on—

1.	£450 in 4 years	at	2	per cent.	per annum.
2.	£325 in $3\frac{1}{2}$ „	„	8	„	„
3.	£700 in 5 „	„	4	„	„
4.	£620 in 2 „	„	$3\frac{1}{2}$	„	„
5.	£180 in 4 „	„	$7\frac{1}{2}$	„	„
6.	£235 in 6 months	„	$3\frac{3}{4}$	„	„
7.	£900 in 3 „	„	10	„	„
8.	£820 in 73 days	„	5	„	„
9.	£750 in 1 month	„	$4\frac{1}{2}$	„	„
10.	£620 in 2 months	„	6	„	„

If we put I for the number of pounds to be paid as interest,

P	„	„	„	lent,
R	„	„	„	to be paid per cent. per annum,
T	„	„	years	for which P is lent,

then I on £100 for 1 year $= R$

$$„ \quad £1 \quad „ \quad „ \quad = \frac{R}{100}$$

$$„ \quad P \quad „ \quad „ \quad = \frac{P \times R}{100}$$

$$„ \quad P \quad „ \quad T \quad = \frac{P \times R \times T}{100}.$$

It will be seen that we might have written at once in the preceding exercise $I = \frac{P \times R \times T}{100}$.

Example.—Find the Simple Interest on £250 at 6 per cent. per annum for 3 years.

$$I = £\frac{250 \times 6 \times 3}{100} = £45.$$

Further Notes

1. When finding the interest for a number of days it is a good plan to express the days as a fraction of a year and then use the formula $I = P \times R \times T \div 100$.

2. When we have to reckon the number of days, say from May 8 to August 26, we omit the first date and add the last. Thus

May	gives	23	days
June	„	30	„
July	„	31	„
August	„	26	„
	Total =	110	„

3. 73 days $= \frac{1}{5}$ of a year; 146 days $= \frac{2}{5}$ of a year.
219 days $= \frac{3}{5}$ „ „ ; 292 days $= \frac{4}{5}$ „ „

4. Always work by decimals, if possible.

Exercise 42

1. Find the total simple interest paid on £736 in 4 years at $7\frac{1}{2}$ per cent. per annum.

2. Find the interest payable on £350 for 7 months at $2\frac{1}{2}$ per cent. per annum.

3. Find the total amount due to be paid after £225 has been lent at $4\frac{1}{2}$ per cent. per annum for 219 days.

4. Find the interest that must be paid half-yearly on a mortgage of £2500 lent at $5\frac{1}{4}$ per cent. per annum.

5. A man insures his house and furniture for £730. He pays 2s. 6d. per cent. per annum. Find his annual payment.

6. What is the simple interest on £80 from 3rd May to 26th September of the same year at $5\frac{1}{4}$ per cent. per annum ?

7. A man invests £7300 from 5th June to 8th August of the same year, and he receives 6 per cent. per annum on his investment. Find the interest.

8. A moneylender charges $12\frac{1}{2}$ per cent. per annum as interest. Find the amount to be repaid on 27th August to clear a debt of £440 contracted on 3rd April.

9. On 6th June a man invests £87 as loan money with a corporation which pays 4 per cent. per annum. The account is closed on 18th August. How much is withdrawn ?

10. A man borrows £5260 when the rate of interest on loans is $4\frac{1}{2}$ per cent. per annum. After 4 months the rate is changed to 5 per cent. per annum, and the man continues to hold the loan till the end of a year from the time when he first borrowed the money. How much interest must he pay for the whole year ?

Example.—Find to the nearest penny the total interest paid on the loan of £350, 11*s.* 0*d. for* 5 *years* 146 *days at* 5 *per cent. per annum if the interest is paid yearly.*

$$\begin{aligned} I &= P \times R \times T \div 100. \\ &= £350{\cdot}55 \times 5 \times 5\tfrac{2}{5} \div 100. \\ &= £1752{\cdot}75 \times 5{\cdot}4 \div 100. \\ &= £9464{\cdot}85 \div 100. \\ &= £94{\cdot}6485. \\ &= £94,\ 12s.\ 11{\cdot}64d. \\ &= £94,\ 13s.\ 0d.,\ \text{to the nearest penny.} \end{aligned}$$

Exercise 43

[Express answers correct to the nearest penny.]

1. Find the total simple interest paid for the loan of £330, 10s. 0d. in 4 years at $5\frac{1}{8}$ per cent. per annum.

2. Find the amount of £314 for 292 days at $3\frac{1}{3}$ per cent. per annum simple interest.

3. Calculate the simple interest on £24, 12s. 0d. from 20th May to 25th December at $2\frac{1}{2}$ per cent. per annum.

4. I invest £715, 8s. 0d. for a period of 146 days at $5\frac{1}{4}$ per cent. per annum. How much do I withdraw when I close the account ?

5. If we can borrow £200 for 1 year and then repay £206, what must we repay for a loan of £28, 18s. 0d. for the same time and at the same rate ?

6. How much simple interest do I draw if I invest £840 at 3 per cent. per annum from 1st February to 7th September 1924 ?

7. Harry's father invests £83 in a bank when the boy is 10 years of age. The money is withdrawn on his 21st birthday. If the bank pays 3 per cent. per annum, how much will be withdrawn (simple interest) ?

8. A tradesman deposits £600 in his bank on 4th July 1926 and draws interest at $2\frac{1}{2}$ per cent. per annum till 17th November 1927. Find the total interest he has received.

9. If in a year £300 amounts to £400, find the amount of £87, 12s. 0d. from 2nd May to 23rd August.

10. If I agree to pay $3\frac{3}{4}$ per cent. as interest on a loan of £832 borrowed on 2nd January, how much do I owe on 24th February ?

Practice

An Aliquot Part is such a part of another quantity as is contained in that quantity an exact number of times, *e.g.* 5s. is an aliquot part of £1, because 5s. is contained in £1 exactly four times.

Practice is the name given to the method of finding the cost, the weight, etc., of a quantity by means of aliquot parts, when the cost, weight, etc., of the unit quantity is stated.

Example (i).—*Find by practice the cost of* 185 *articles at* 16*s.* $10\frac{1}{2}$*d. each.*

				£	s.	d.
Cost of 185 at		£1	each	=185	0	0
,,	,,	10s.	,,	= 92	10	0
,,	,,	6s. 8d.	,,	= 61	13	4
,,	,,	2d.	,,	= 1	10	10
,,	,,	$\frac{1}{2}$d.	,,	= 0	7	$8\frac{1}{2}$
,,	,,	16s. $10\frac{1}{2}$d.		=156	1	$10\frac{1}{2}$

Be on the look-out for short methods, *e.g.*—

Example (ii).—*Find the cost of* 750 *coats at* 9s. 11*d. each.*

				£	s.	d.
Cost of 750 coats at	£1	each	=	750	0	0
,, ,,	10s.	,,	=	375	0	0
,, ,,	1d.	,,	=	3	2	6
,, ,,	9s. 11d.	,,	=	371	17	6

Example (iii).—*Find the cost of* 5964 *lbs. of yarn at* 1s. $7\frac{15}{16}$*d. per lb.*

				£	s.	d.
Cost of 5964 lbs. at	1s.	per lb.	=	298	4	0
,, ,,	6d.	,,	=	149	2	0
,, ,,	1d.	,,	=	24	17	0
,, ,,	$\frac{8}{16}$d.	,,	=	12	8	6
,, ,,	$\frac{4}{16}$d.	,,	=	6	4	3
,, ,,	$\frac{2}{16}$d.	,,	=	3	2	$1\frac{1}{2}$
,, ,,	$\frac{1}{16}$d.	,,	=	1	11	$0\frac{3}{4}$
,, ,,	1s. $7\frac{15}{16}$d.	,,	=	495	8	$11\frac{1}{4}$

Exercise 44

Find by aliquot parts the prices of

1. 583 plants at (*a*) 4s. 6d., (*b*) 9s. 6d., (*c*) 18s. 6d.
2. 375 fountain pens at (*a*) £1, 3s. 6d., (*b*) £5, 13s. 4d., (*c*) £3, 8s. 9d.
3. 485 suits at (*a*) £6, 2s. 1d., (*b*) £7, 2s. 8d., (*c*) £4, 5s. 6d.
4. 396 tables at (*a*) 17s., (*b*) 14s., (*c*) 19s. 6d.

Work by the subtraction method the prices of

5. 28 chairs at (*a*) $11\frac{1}{2}$d., (*b*) 3s. $5\frac{3}{4}$d., (*c*) 4s. $11\frac{3}{4}$d.
6. 352 cupboards at (*a*) 18s. $11\frac{1}{2}$d., (*b*) £3, 19s. 9d., (*c*) 5s. $5\frac{3}{4}$d.
7. 497 fenders at (*a*) 18s. $5\frac{3}{4}$d., (*b*) 17s. 11d., (*c*) 19s. 5d.
8. 654 grates at (*a*) £1, 5s. 11d., (*b*) £4, 13s. 11d., (*c*) £1, 10s. 11d.

Find the cost of

9. 9 tons 10 cwts. of coal at £2, 7s. 10d. per ton.
10. 17 tons 15 cwts. of metal at £19, 19s. 6d. per ton.
11. 5 tons 2 cwts. 3 qrs. of butter at £6, 10s. 0d. per cwt.
12. 17 cwts. 3 qrs. 21 lbs. of pulp at £2, 10s. 0d. per ton.

13. 4 miles 4 fur. of railing at £5, 5s. 0d. per mile.
14. 7 miles 7 fur. 20 poles of fencing at £14 per mile.
15. 8 miles 3 fur. 25 poles of draining at £18 per fur.
16. 5 fur. 110 yds. of netting at £10, 13s. 4d. per mile.
17. 5 galls. 2 qts. 1 pt. of cream at 7s. a gall.
18. 7 galls. 3 qts. 1 pt. of oil at 5s. $4\frac{1}{2}$d. a qt.
19. 15 qts. 1 pt. of petrol at 2s. 6d. a gall.
20. 780 lbs. of yarn at (*a*) 1s. $9\frac{11}{16}$d., (*b*) 1s. $8\frac{13}{16}$d., (*c*) 2s. $4\frac{15}{16}$d., per lb.

Exercise 45

[To be worked by short methods.]

	(*a*)	(*b*)	(*c*)
1.	7846×99;	3592×999;	7148×199.
2.	13595×25;	16732×250;	85724×2500.
3.	$5641 \div 25$;	$41389 \div 125$;	$76532 \div 1250$.
4.	735×625;	$38412 \div 62\frac{1}{2}$;	$85155 \div 16\frac{2}{3}$.
5.	239 times $4\frac{1}{2}$d.;	246 times $7\frac{1}{2}$d.;	241 times $8\frac{1}{4}$d.
6.	$5{\cdot}82 \times 100$;	$7423 \div 1000$;	$3841 \div 200$.
7.	$80 - 8{\cdot}0$;	$100 - 10{\cdot}0$;	$11 - 1{\cdot}1$.

8. (*a*) 15 pictures at £3, 10s. each; (*b*) 334 books at 1s. 8d. each.
9. (*a*) 12 pens at 3s. $6\frac{1}{2}$d. each; (*b*) 12 hats at 5s. $5\frac{1}{2}$d. each.
10. (*a*) 1 article at 11s. 6d. a dozen; (*b*) 1 article at £3, 15s. 9d. a dozen.
11. (*a*) 13 boxes at 4s. $4\frac{1}{2}$d. each; (*b*) 11 knives at 7s. $5\frac{1}{2}$d. each.
12. (*a*) 20 packets at 4s. 5d. each; (*b*) 21 caps at 6s. 11d. each.
13. (*a*) 240 tins at $3\frac{1}{2}$d. each; (*b*) 240 badges at $5\frac{1}{2}$d. each.
14. (*a*) 241 lemons at $2\frac{1}{2}$d. each; (*b*) 239 tiles at $7\frac{1}{2}$d. each.
15. (*a*) 480 stones at $11\frac{1}{2}$d. each; (*b*) 480 dolls at 1s. $1\frac{1}{2}$d. each.
16. (*a*) 483 toys at $6\frac{1}{2}$d. each; (*b*) 478 ties at $8\frac{1}{2}$d. each.
17. (*a*) 960 toys at 7d. each; (*b*) 960 reels at 4d. each.
18. (*a*) 961 cakes at $1\frac{1}{2}$d. each; (*b*) 958 loaves at $5\frac{1}{2}$d. each.

Invoices

A Specimen Invoice

Telephone No. 760.	78, Broad Street,
Telegraphic Address:	Leamington.
"*Dudell, Leamington.*"	1*st March*, 1926.

Messrs Arrow & Co., Ltd.

Bought of C. M. Dudley & Sons, *Grocers.*

Terms $2\frac{1}{2}$% *monthly.*

				£	s.	d.	£	s.	d.
15 kegs .	Lard, each 48 lbs.	at	9d. per lb.	27	0	0			
200 lbs. .	Butter . .	at	1s. 6d. per lb.	15	0	0			
2 cwts. .	Sago . .	at	39s. 8d. per cwt.	3	19	4			
$\frac{1}{2}$ cwt. .	Cheese . .	at	1s. 2d. per lb.	3	5	4			
							49	4	8

Exercise 46

Make out invoices for the following goods.

Assume that you are the seller, use your own address and the current date, and invent (i) a telephone number, (ii) a telegraphic address, and (iii) the names of purchasers.

1. 60 readers at 2s. $4\frac{1}{2}$d. each.
   3 gross writing books at $3\frac{1}{2}$d. each.
   2 gross pencils at $2\frac{1}{2}$d. each.
   30 boxes pens at $7\frac{1}{2}$d. a box.
2. 500 marbles at 25 for $2\frac{1}{2}$d.
   200 tops at $1\frac{1}{2}$d. each.
   30 bats at 12s. $7\frac{1}{2}$d. each.
   50 balls at 2s. 11d. each.
3. 30 lbs. raisins at $6\frac{1}{2}$d. per lb.
   $3\frac{1}{2}$ doz. eggs at 8 for a shilling.
   $2\frac{1}{4}$ lbs. butter at 2s. 3d. a lb.
   21 lbs. apples at 4s. 8d. per stone.
   $\frac{3}{4}$ lb. tea at 2s. 8d. per lb.
4. 2 doz. cakes at 4 for $5\frac{1}{2}$d.
   7 loaves at $5\frac{1}{2}$d. each.
   $\frac{3}{4}$ doz. tins of fruit at $11\frac{1}{2}$d. each.
   8 pies at $7\frac{1}{2}$d. each.
5. $3\frac{1}{4}$ tons tallow at 11s. 3d. per cwt.
   19 cwts. wax at £1, 1s. 8d. per ton.
   $\frac{1}{2}$ keg of grease at 17s. 8d. per keg.
   $\frac{1}{2}$ cwt. soap at $5\frac{1}{2}$d. per lb.
   16 galls. oil at $7\frac{1}{2}$d. per quart.
6. 8000 fine bricks at £1, 2s. 6d. per 100.
   1200 tiles at $1\frac{1}{2}$d. each.
   1500 perforated bricks at 48s. for 50.
   100 glazed bricks at 3s. 6d. per doz.
7. 12 ozs. leather at 2s. 8d. per lb.
   $2\frac{1}{2}$ ozs. brass nails at 2s. per lb.
   4 ozs. steel nails at 6d. per lb.
   2 sticks cobbler's wax at 8 for 1s.
8. 50 oranges at 4 for 6d.
   $2\frac{1}{2}$ doz. lemons at 4 for 5d.
   17 bananas at 2 for $2\frac{1}{2}$d.
   12 boxes dates at 1s. $1\frac{1}{2}$d. per box.

To encourage prompt payment of an account a trader often makes a deduction from the total money he is entitled to receive. This deduction is known as *discount*, and is usually a percentage of the total money due. Discount is generally allowed on complete shillings only, and is corrected to the nearest penny.

Example of Invoice with discount.

Terms $2\frac{1}{2}\%$ monthly.

			£	s.	d.	£	s.	d.
38 lbs.	of currants at	8d. per lb.	1	5	4			
12 „	of figs at	9d. „	0	9	0			
						1	14	4
		Discount				0	0	10
						1	13	6

Exercise 47

Make out and receipt the following invoices after allowing the discount specified :—

1. 1 piano at £59, 10s. 0d.
   4 chairs at £2, 18s. 0d. each.
   Bookcase, 7 sections, at £1, 1s. 6d. per section.
   3 carpets at £11, 19s. 0d. each.
   Discount, $2\frac{1}{2}\%$.

2. 56 lbs. soap at $7\frac{1}{2}$d. per lb.
   156 candles at 1s. 1d. per doz.
   18 qts. best oil at 1s. 2d. per gall.
   $4\frac{1}{2}$ galls. oil at $8\frac{1}{2}$d. per gall.
   Discount, 3%.

3. 400 pairs boots at 28s. $3\frac{1}{2}$d. per pair.
   172 pairs slippers at 4s. 9d. per pair.
   29 pairs leggings at 12s. 4d. per pair.
   13 doz. soles at £9, 18s. per gross.
   Discount, $3\frac{1}{4}\%$.

4. $6\frac{1}{2}$ doz. eggs at 8 for 1s.
   $7\frac{1}{2}$ lbs. butter at 2s. 3d. per lb.
   $3\frac{3}{4}$ lbs. ham at 1s. 2d. per lb.
   5 qts. 1 pt. milk at $2\frac{1}{2}$d. per pt.
   4 pots cream at 1s. $1\frac{1}{2}$d. per pot.
   Discount, 1d. in the shilling.

5. 7800 cu. ft. gas at 4s. 2d. per 1000 cu. ft.
   Rent of meter for 7 months at 7s. per annum.
   Rent of meter for 5 months at 8s. per annum.
   12 hrs. workman's time at 1s. $1\frac{1}{2}$d. per hour.
   Discount, 3s. in the £.

6. Coal deliveries :
   3 Jan., 14 cwts. at 2s. $1\frac{1}{2}$d. per cwt.
   8 March, $\frac{1}{4}$ ton at 2s. 3d. per cwt.
   19 May, 23 cwts. at 2s. 1d. per cwt.
   Discount, $2\frac{1}{2}\%$.

7. Joiners' services :
   48 hrs. at 1s. $9\frac{1}{2}$d. per hr.
   7 hrs. travelling time at $10\frac{1}{2}$d. per hr.
   3 hrs. overtime at 2s. $2\frac{1}{2}$d. per hr.
   Discount, $2\frac{1}{2}\%$.

8. 19 rolls wall-paper at 2s. $1\frac{1}{2}$d. per roll.
   5 rolls ceiling-paper at 1s. $2\frac{1}{2}$d. per roll
   41 yds. frieze at 9d. per yd.
   Time, 26 hrs. at 1s. 4d. per hr.
   Discount, 2d. in the shilling.

9. 3 gross bananas at 2 for $2\frac{1}{2}$d.
   18 lbs. plums at 1s. 9d. per stone.
   3 doz. oranges at 16 for 1s.
   8 lbs. dates at $4\frac{1}{2}$d. per lb.
   25 lbs. potatoes at 5 lbs. for $7\frac{1}{2}$d.
   Discount, 5%.

10. 340 yds. boarding at 4d. per foot.
    8 yds. cord at $4\frac{1}{2}$d. per yd.
    3 men each 5 hrs. at 1s. 1d. per hr.
    Travelling time for each, 2 hrs. at 7d. per hr.
    Discount, $3\frac{1}{4}\%$.

Average

To find the average of a number of quantities we must (i) find the sum total of the quantities, and (ii) divide this by the number of the quantities.

If the quantities are a, b, c, d, the average $= \dfrac{a+b+c+d}{4}$.

Example.—Find the average length of four rods measuring respectively 3′ 6″, 4′ 8″, 2′ 10″, 1′ 11″.

The average $= \dfrac{3'\,6'' + 4'\,8'' + 2'\,10'' + 1'\,11''}{4} = \dfrac{12'\,11''}{4} = 3'\,2\tfrac{3}{4}''$.

EXERCISE 48

1. The average amount of money in 20 bags is £3, 5s. 6d., and the average amount in 10 of the bags is £2, 19s. 6d. What is the average amount in the remaining 10 bags ?

2. The average cost of a dozen fountain pens is 18s. 6d. The total cost of 3 of them is £3 ; the cost of another 3 is 10s. 6d. each ; the cost of another 3 is a guinea each. Find the average cost of the remaining 3.

3. The average weight of 3 trucks of coal is 5 tons 13 cwts. The weight of 1 truck is $4\frac{3}{4}$ tons. Find the average weight of the other two.

4. When 20 lbs. of tea at 3s. 4d. per lb. are mixed with 15 lbs. at 2s. 6d. a lb., find the least price per lb. at which the mixture may be sold so as to make a total profit of £1 on the sale of the whole.

5. Find the average result obtained by three boys working an experiment. The first boy's result was 18·55 grams, the second 18·57 grams, and the third 18·50 grams. Which bo is nearest to the average result ?

6. The runs made by a cricketer in twelve matches are respectively : 5, 9, 4, 0, 20, 70 (not out), 15 (not out), 7, 8, 3, 22, 0. Find his average runs per innings.

7. The daily readings of a thermometer at 9 a.m. for a week were : 49° F., 51° F., 50·5° F., 54° F., 45·5° F., 48·5° F., 58·6° F. Find the average of these readings correct to the second decimal place.

8. A grocer's takings for five days are respectively £10, 10s. $10\frac{1}{2}$d. ; £12, 15s. $5\frac{1}{2}$d. ; £7, 17s. $4\frac{1}{2}$d. ; £12, 16s. $11\frac{1}{2}$d. ; and £13, 14s. 9d. How much must he take on the sixth day to make his average for the six days exactly £14 a day ?

9. There are 30 pupils in a class. Ten pupils work 4 sums correctly, 9 work 3 correctly, 6 work 2 correctly, 2 work 1 correctly, and the remainder work none correctly. What is the average number of sums worked correctly by the class?

10. If the price of coal in the month of July is 44s. per ton, and if it rises each successive month 1s., what is the average price for the half-year?

SECTION III

ALGEBRA

First Steps

A pile of six books contains a definite number of books, namely, 6, but a pile containing *some* books is only one of an unlimited number of such piles.

Some books is an *indefinite* quantity of books, and may be represented by various symbols, *e.g.* x books, y books, and so on.

We follow the same laws in using symbols as we do in arithmetic.

Thus : $\quad$ 6 men + 4 men = (6 + 4) men = 10 men.
And $\quad$ x men + y men = $(x+y)$ men.

Notice the brackets. We do not say $x+y$ men, for this means x of something unknown plus y men.

Also $\quad$ 5 horses − 3 horses = (5 − 3) horses = 2 horses.
And $\quad$ a horses − b horses = $(a-b)$ horses.

Exercise 49

[Be very careful to write the answers correctly.]

1. If there are m tops in one box, n tops in another, and y tops in another, how many tops are there in the three boxes ?

2. If Mary had 20 pennies in her pocket and then gave away b pennies, how many pennies would she have left ?

3. If 16 boxes were stored in a room, and then g boxes added and h boxes removed, how many remained in the room ?

4. If Dan had h shillings and Fred k shillings, how much money had they both remaining if each gave away 2 shillings ?

5. If Tom had x pennies and John had 2 pennies more than Tom, how many had they both together ?

6. When we add a to b and take away c, what remains ?

7. Write a statement showing that x is greater than y by z.

8. Explain the statement, $r=s-t$.

9. The figure 5 represents $1+1+1+1+1$. How often must 1 be written to represent x units ?

10. A sea-wall is built in two sections ; the foundation contains x bricks and the exposed portion y bricks. If the sea washes away p bricks, how many remain ?

11. If the distance between the lines in an exercise book is p ins., what is the distance from the first to the last line on a page of 20 lines ?

12. From London to Paris is $(2q+c)$ miles as the crow flies. How far from London is an aeroplane which has travelled q miles towards London from Paris ?

When a man has travelled four times over a distance of x miles, we may say that he has travelled $4x$ miles.

Note that there is no sign between the 4 and the x to show that we mean $4\times x$; $4x=4\times x$ or $x\times 4$.

When the journey has been made n times the distance travelled is nx miles.

So also, $x\times y\times z=xyz$, or xzy, or yzx, or yxz, or zxy, or zyx.

Just as a fifth part of 6 miles is $\frac{6}{5}$ mile, so a distance of x miles divided by $5=\frac{x}{5}$ miles, and if divided into p equal parts, each part$=\frac{x}{p}$ miles.

Notice that it is possible to cancel symbols just as we cancel numbers, *e.g.*, $ax\div ay=\frac{\cancel{a}x}{\cancel{a}y}=\frac{x}{y}$.

Exercise 50

1. The sides of an equilateral triangle are each $2x$ ins. long. What is the length of the perimeter ?

2. The sides of a square field are each $5x$ yds. in length. Find the number of yards a man has travelled when he has walked round this field 20 times.

Now express the answer in miles.

3. The sum of 12 equal numbers is $36m$. What is each number ?

4. If 5 eggs cost a shilling, how many can be bought for z pence ?

5. How many pence are there in a shillings and b pence ?

6. Express x pounds$+y$ shillings$+z$ pence in pence.

7. If r pounds$+r$ shillings are divided equally among t boys, how many shillings does each boy receive ?

8. Express a yds.$+b$ ft.$+c$ ins. in inches.

9. Write a statement showing that m is to be multiplied by n and the product taken away from p.

10. When x is divided by y and z added to the quotient, what is the result ?

Formulæ and Equations

We have seen that the area of any rectangle may be found by multiplying the units in the length of the base by the similar units in the altitude. We may express this by symbols thus: when B=base and H=altitude, then the area=BH.

This statement is called a *formula* for the area of a rectangle.

When considering a particular rectangle, B and H will have particular values, but the above statement is true whatever the base and the altitude may be, *i.e.* it is a *general statement* for the area of a rectangle.

Other formulæ we know are: Area of a square=BB or B^2; Perimeter of a rectangle=2B+2H, or 2(B+H).

When we wish to evaluate a formula we *substitute* particular values for the symbols, thus: If in the formula for a rectangle, B=4 ins. and H=3 ins., then the area=BH=(4×3) sq. ins. =12 sq. ins.

Exercise 51

1. Draw a line $2\frac{1}{2}$ ins. long. If one portion of this line is x ins. long, what is the length of the other portion ?

2. This line A————B is m ins. long. Draw lines $2m$ ins., $3m$ ins., $\frac{m}{2}$ ins., $\frac{3m}{2}$ ins. in length.

3. If $x=5$, $y=6$, $z=7$, find the value of (i) xy, (ii) $x+y+z$, (iii) $x+y-z$, (iv) $z-x$, (v) $\frac{yz}{x}$, (vi) $\frac{xyz}{z}$.

4. How many feet are there in (i) x yds., (ii) $(x+2)$ yds., (iii) $7x$ ins., (iv) $36x$ ins., (v) $3xy$ ins., (vi) $40x$ yds. ?

5. Express (i) x yds.$+y$ ft. in feet; (ii) x pounds, y shillings, z pence in farthings.

6. From the formula for the perimeter of a rectangle, find

the perimeter of a rectangular room when the length is 18 ft. and the breadth 14 ft.

7. Find how many shillings are left out of x shillings after buying y lbs. of rice at z pence per lb.

8. Find, by using formula, the area of a rectangle which has a base of $2a$ ins. and an altitude of $4b$ ins.

9. One tap fills a tank at the rate of x galls. per hour, a second tap fills it at the rate of y galls. per hour, a third tap drains the tank at the rate of z galls. per hour. Write a formula to show the quantity of water in the tank when all three taps run for n hours.

10. A box of dominoes weighs x ozs. The box alone weighs p ozs., and each domino q ozs. Write a formula to show the number of dominoes in the box. Substitute in the formula when $x=12$, $p=5$, $q=\frac{1}{4}$, and find the number of dominoes.

When we are told that 8 times a certain number is 56, we may write the statement thus: $8 \times$ an unknown number$=56$. Putting the symbol x for the unknown number we obtain: $8x=56$.

This statement is spoken of as an *equation*, because the two sides, $8x$ and 56, are equal.

When we find the value of x, we are said to *solve* the equation for x, thus:

$$\text{When } 8x=56, \text{ then } x=\frac{56}{8}=7.$$

Example (i).—*Find the value of* q *when* q$-7=21$.

$$\text{If } q-7=21, \text{ then } q-7+7=21+7. \quad \therefore q=28.$$

Example (ii).—*Find the value of* t, *when* t$+8=16$.

$$\text{If } t+8=16, \text{ then } t+8-8=16-8. \quad \therefore t=8.$$

Example (iii).—*If I think of a certain number, double it, and add* 7, *I find the result is* 43. *What is the certain number?*

Let $x=$the certain number.

Then $2x+7=43$.

$\therefore \quad 2x+7-7=43-7.$

$\therefore \quad 2x=36.$

$\therefore \quad x=\frac{36}{2}=18.$

Exercise 52

1. Find the value of the unknown in each of the following :—

(i) $13x=65$, (ii) $\frac{3}{4}x=27$, (iii) $x-17=38$, (iv) $y+14\frac{1}{2}=26\frac{3}{4}$, (v) $2y-7=18$.

2. Solve the following for x :—

(i) $12x-3=45$, (ii) $\frac{1}{2}x+6=30$, (iii) $\frac{x}{10}=\frac{9}{10}$, (iv) $\frac{x}{5}=\frac{4}{10}$.

3. Find what b stands for if

(i) $\frac{5}{b+2}=\frac{1}{2}$, (ii) $\frac{4}{7}=\frac{8}{b}$, (iii) $\frac{b}{3}=\frac{1}{2}$.

4. If a man owned three times as many sheep as there were in one of his fields and afterwards 4 sheep died, he would have 23. How many sheep were there in the field ?

5. The area of a rectangle is $29\frac{1}{2}$ sq. ft. ; its base is x ft. and its altitude 2 ft. Find the length of its base in feet.

6. In p innings a cricketer makes 320 runs. If his batting average is $45\frac{5}{7}$ runs, how many innings did he play ?

7. A tennis ball weighs x grams. If a man puts 5 tennis balls and a 2-gram weight on one side of a balance, the total weight is 327 grams. Find the weight of a tennis ball.

8. If x is 3 less than y, find the value of x when $5x+2y=29$.

9. When $8m$ is greater than $3m$ by 35, what is the value of m ?

10. The difference between one-third of a certain number and a quarter of it is 8. Find the number.

Some Definitions

When we have quantities connected by the sign + or by the sign −, each of these quantities is spoken of as a *term*, and the combination of terms is called an *expression*.

Thus, $a+5b-4c$ is an expression of three terms.

The terms a and $5b$ are *positive* terms, and $-4c$ is a *negative* term. The first term is always a positive term unless preceded by a minus sign.

When a term is itself the product of two or more factors, then any one of these factors or the product of two or more of

them is known as the *coefficient* of the remaining factors. Thus $5abc$ is a term made up of the factors $5, a, b, c$.

$\therefore$ 5 is a coefficient of abc.
$\therefore a$,, ,, $5bc$.
$\therefore ab$,, ,, $5c$, and so on.

5 is known as the *numerical coefficient.*

If the numerical coefficient is not expressed it must be understood to be 1, thus: $xy=1xy$; $mnp=1mnp$. There *must* be a numerical coefficient.

Consider the expression

$$5ab+3ab+7ab+2cd-4ab.$$

$5ab$, $3ab$, $7ab$, and $-4ab$ are terms which only differ in their numerical coefficient. They are said to be *like terms.* $5ab$ and $2cd$ do not contain the same combination of symbols. They are said to be *unlike terms.*

$a\times a$ may be written a^2; $a\times a\times a$ may be written a^3. $a\times a\times a\times a$ may be written a^4, and so on. The number in small print written above and to the right of the symbol is known as the *index* (plural, *indices*) of the symbol. If no index is written it is understood to be 1, thus: $a=a^1$. a^1, a^2, a^3, a^4 . . . are said to be the 1st, 2nd, 3rd, 4th . . . *powers* of a, respectively.

a^2 is also known as the square of a or a squared.

a^3 is also known as the cube of a or a cubed.

But a^4 is always known as the fourth power of a or a to the fourth.

[*Note.*—The index tells the number of equal factors which, multiplied together, will produce the required power. Thus a^n means the quantity obtained by multiplying n factors each equal to a.]

The *square root* of a number is the number which, when squared, produces the original number. The sign $\sqrt{\ }$ is used to denote a square root.

Thus $\sqrt{16}$ denotes the square root of 16 (*i.e.* 4, for $4^2=16$).

Consequently $\sqrt{16}\times\sqrt{16}=16$ (for $4\times4=16$).

The *cube root* of a number is the number which, when cubed, gives the original number. The sign $\sqrt[3]{\ }$ is used to denote a cube root. Thus $\sqrt[3]{64}$ denotes the cube root of 64 (*i.e.* 4, for $4^3=64$).

Consequently $\sqrt[3]{64}\times\sqrt[3]{64}\times\sqrt[3]{64}=64$ (for $4\times4\times4=64$).

We may express the fourth, fifth, etc., root in a similar way They are denoted by signs $\sqrt[4]{\ }$, $\sqrt[5]{\ }$, etc.

[*Note.*—$\sqrt[n]{a}$ means a number which, if taken as a factor n times, produces a.]

Negative Quantities

The formula to express a shopkeeper's profit is $P=I-E$, when P represents profit; I, income; and E, expenditure.

(i) If $I=£8$, and $E=£3$, then $P=£\,(8-3)=£5$.

(ii) If $I=£8$, and $E=£8$, then $P=£\,(8-8)=£0$.

(iii) If $I=£8$, and $E=£12$, then $P=£\,(8-12)=$?

In this third case the profit is £4 less than nothing, and is written £ (-4). -4 is said to be a negative quantity, which may always be recognised by the preceding sign, $-$.

The pupil will be familiar with such statements as $-10°$ F., $-5°$ C.

In business a man in debt may be said to be worth (say) £-200, £-500, or £-1000.

Graphical Illustration.—From a point O draw a line of any length to the right, say to R. Now produce this line to the left, say to L. If O is the zero point, the direction OR is regarded as the positive direction and OL as the negative direction.

If fig. 4 were a scale drawing of a railway track, and a train ran from O to R, we say it ran 4 miles in a positive direction; and if it then returned from R to O, we say it ran 4 miles in a negative direction. Its distance from O was then $(+4-4)$ miles or $+4$ miles -4 miles, which is nil.

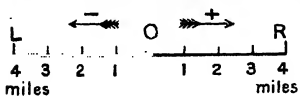


Fig. 4.

If the train ran from O 3 miles towards L and then 2 miles towards R, its distance from O would be -3 miles $+2$ miles, *i.e.* -1 mile from O.

Starting from the zero point and going m miles towards R, the train would be $+m$ miles from O. On returning b miles towards L, it would be $(m-b)$ miles from O.

Exercise 53

1. A man walks from a town A. He walks x miles to the west, then y miles to the east, and takes a train p miles to the east. Write a formula to show his position in relation to A.

2. A crane holds a block of stone f yds. above the water of a lake. The block is lowered m ft. How far is it now from the water surface ? If the lake is q yds. deep, how far must the block be further lowered to reach the bed of the lake ? Make a sketch.

3. A clock which has gained x minutes has its minute hand pushed back $3x$ minutes. How many minutes fast is the clock now ?

4. A grocer starts with x lbs. of flour in a sack. He serves m customers with b lbs. of flour each, adds n lbs. of flour, and then serves y lbs. of flour. How much remains?

5. What is the meaning of

(*a*) A man's income is £-8 per month.
(*b*) A tree top is -3 ft. above a house window.
(*c*) A man gives his employer £-7.
(*d*) The bottom of a lake is -380 ft. above sea level.

6. What is the profit made by a builder who builds a house for £$12ab$ and sells it for £$9ab$?

7. A clock gains $-4a$ hours in the day and loses b hours in the night. How much fast is it after 24 hours?

8. A flight of stairs has steps each of u ins. rise. What height from his starting-point is a man who mounts 12 steps, descends 18 steps, and mounts 4 steps?

9. A flag-pole is 34 ft. in height. A flag is $-a$ ft. above the top of the pole. How much must the flag be raised so as to be 2 ft. from the ground?

10. A tourist starts on a mountain tramp from a town B, which is marked on the map on the 350-ft. contour line. The tourist ascends 820 ft., descends b ft., and then ascends c ft. If the final position of the tourist were marked on the map, on which contour line would it be placed?

11. In collecting penny fares on a tramway route one conductor draws £x, another £x, 10s. 0d., and a third $4x$ shillings. Find the total number of penny tickets issued.

12. From a tank containing $12m$ galls. a milkman fills 3 cans each holding g pts.; and then draws $12g$ pts. How much remains in the tank?

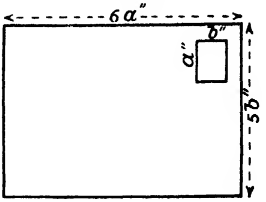


Fig. 5.

13. The area of a plate is $(8a^2+12b^2)$ sq. ins. If 8 square plates having sides a ins. long, and 3 square plates having sides b ins. long, are cut out, what area remains?

14. A fruit-grower gets t lbs. of apples from one orchard, $4s$ lbs. from another, and $40t$ lbs. from a third. He rejects $(2s+t)$ lbs.; what is the weight of his crop?

15. The dimensions of a postcard and stamp are as shown

in fig. 5. What area remains uncovered when 3 stamps are gummed on the postcard ?

Simplification

An expression is *simplified* by collecting like terms,

e.g. $$2x+3x+5x=(2+3+5)x=10x$$
$$7ab-3ab \quad =(7-3)ab=4ab.$$

In the expression $5lm-6lm+8lm-3lm$, all the terms are *like*, but some are positive and some negative terms. Collecting positive terms we obtain $5lm+8lm=13lm$.

Collecting negative terms we obtain $-6lm-3lm=-9lm$.

$$\therefore \text{ Expression}=5lm+8lm-6lm-3lm$$
$$=13lm-9lm$$
$$=4lm.$$

In the expression $3x-2y+2x+3y-3y-4z$ all the terms are not like terms.

First collect all like terms and arrange thus :

$$\text{Expression}=3x+2x-2y+3y-3y-4z.$$

Now simplify the like terms in succession.

$$\text{Expression}=5x-2y-4z.$$

Exercise 54

Simplify

1. $7a+3a+2a+8a.$
2. $6a-8b+3a-5b-7a+6b.$
3. $4bc-6bc+8a-2bc.$
4. $a+2d-5a+6d-5ad.$
5. $3x(a+b)+2x(a+b)-4x(a+b).$
6. $9x-3x+\frac{1}{2}x-4\frac{1}{2}x.$
7. $5x-6y+7z-2x-4y-5z.$
8. $a-b-c+7+3a+2b+3c-8.$
9. $a(b-c)+3a(b-c)-7a(b-c).$
10. $5r+4s-6r-5s+4r-5s+2r.$

Addition

Example.—Add $5a+3b-5c$, $4a-2b+6c$, $-a-b+c$, *and* $9a-10b-11c$.

Arrange the like terms in columns, thus :

$$\begin{array}{r} 5a+\ 3b-\ 5c \\ 4a-\ 2b+\ 6c \\ -a-\ \ b+\ \ c \\ 9a-10b-11c \\ \hline 17a-10b-\ 9c \\ \hline \end{array}$$

Deal with each column in turn and simplify, thus :

$$(5+4+9-1)a=(18-1)a=17a$$
$$(3-2-1-10)b=(3-13)b=-10b$$
$$(6+1-5-11)c=(7-16)c=-9c.$$

Notes.—1. These simplifications should be done mentally.

2. $17a$ is said to be the ***algebraic sum*** of $5a$, $4a$, $-a$, and $9a$, even though one of the terms is a negative term.

Exercise 55

Add together :

1. $abc+by-bx$, $3abc-5by+3bx$, $7by-4abc-bx$.
2. $3ab-4ac+4ad$, $7ac-3ad+2ab$, $7ad-3ac-4ab$, $-ab-ac-ad$.
3. $5rs+6rl-7t$, $7rl+5t-9rs$, $8rl+5t+6rs$, $rs+rl+t$.
4. $kr-rs-st$, $4kr-4kr-4st$, $3st+7kr-9rs$, $8rs-3kr-st$.
5. $xy-xz+yz$, $2xy-2xz+3yz$, $5xy+2xz+9yz$, $10xy-10xz-10yz$.
6. $-5x+7y+4z$, $12x-12y-12z$, $x+y+z$.
7. $(a+b+c)$, $-(a-b-c)$, $(-a+b-c)$, $-(a-b+c)$.
8. $5(x+y+z)$, $-2(x+y+z)$, $-(a+b+c)$, $2(a+b+c)$.
9. $12a-6b$, $-a+6b+4c$, $-3a-3b+5c$.
10. $-4x(m-n+p)$, $-x(m-n+p)$, $+8x(m-n+p)$, $-3x(m-n+p)$.

Subtraction

Consider a boy who has some marbles, say 10. As he gives them away, one at a time, he has successively 9, 8, 7, 6, 5, 4, 3, 2, 1, 0. If he borrows 1 marble and loses it he owes 1. We have learned to call his debt a negative quantity, in this case -1. The result is $10-11=-1$.

If a boy owes 8 marbles and wins 20 marbles, he then owns $-8+20=12$ marbles. If he owes 6 marbles and loses 20, the result is he owns $-6-20=-26$ marbles.

Suppose a boy started a game with 4 marbles. We may say:

	Marbles.		Marbles.		Marbles.
Starting with	4	and losing	4	he has left	0
"	4	"	3	"	1
"	4	"	2	"	2
"	4	"	1	"	3
"	4	"	0	"	4
"	4	"	-1	"	5
"	4	"	-2	"	6
"	4	"	-3	"	7

When he loses -1 marble (*i.e.* one less than zero) he really gains 1 marble. When he loses -2 marbles he gains 2 marbles, and so on.

	Marbles.		Marbles.		Marbles.
Again, starting with	-4	and losing	4	he has left	-8
"	-4	"	3	"	-7
"	-4	"	2	"	-6
"	-4	"	1	"	-5
"	-4	"	0	"	-4
"	-4	"	-1	"	-3
"	-4	"	-2	"	-2
"	-4	"	-3	"	-1

Losing has the same effect as "taking away" or subtracting.

Selecting four typical results from the above, we find:

$$4-(+2)=2$$
$$4-(-2)=6$$
$$-4-(+2)=-6$$
$$-4-(-2)=-2.$$

Using symbols, we see:

$$4a-(+2a)=2a$$
$$4a-(-2a)=6a$$
$$-4a-(+2a)=-6a$$
$$-4a-(-2a)=-2a.$$

Now note that if we *change the sign of each of the quantities to be subtracted and then proceed to find the algebraic sum*, we obtain the same results.

Test this rule in every case above.

Example.—From $4x-5y+6z$ *take* $3x+2y+6z$.

Collect the like terms in columns and express thus:

$$\begin{array}{r} 4x-5y+6z \\ \underline{3x+2y+6z} \\ \underline{\underline{x-7y}} \end{array}$$

Working.—Remembering the above rule, proceed as follows: Starting on the left, change $3x$ to $-3x$ (do not write this; think it). Then proceed, $4-3=1$. $1x$ is written x. Next, change $+2y$ to $-2y$. Then $-5y-2y=-7y$. Next, change $+6z$ to $-6z$. Then $+6z-6z=0$. (Do not write 0. Leave a blank as shown.)

Exercise 56

1. From 8 take -3, from $4x$ take $3x$, from $3y$ take $-2y$, from $-2z$ take $-2z$.
2. Take $-2a^2$ from $-8a^2$.
3. From $2x-3xy-3z$ take $-2x-2xy+3z$.
4. From $a^2-2ab+b^2$ take $3a^2-3ab-3b^2$.
5. From $3m^2-4mn-n^2$ take $m^2-2mn-2n^2$.
6. From $4l-3m-2p-r$ take $2m+2n+2p+2r$.
7. From $a^2-2az+z$ take $2a^2-2az+z$.
8. From $x^2+2bx+3ax$ take $2x^2-2bx-6a$.
9. Take $6p-3q+5r$ from $2p-5q+6r$.
10. Take $2a^2-3b^2+5ab$ from $5a^2-b^2-2ab$.

Brackets

Sometimes we find quantities used with brackets, and sometimes we wish to enclose quantities in brackets.

There are 4 kinds of brackets used, namely: (i) *the vinculum,* used thus, $\overline{5x+3}$; (ii) *the small bracket,* used thus, $(5x+3)$; (iii) *the bracket,* used thus, $\{2-(5x+3)\}$; and (iv) *the large bracket,* used thus, $[5+\{2-(5x+3)\}]$.

This order should be remembered, because, when we are simplifying an expression and wish to remove the brackets used, we must remove them one at a time and *in the order shown.*

Remember that when a minus sign precedes a bracket it means that all the quantities within the bracket are to be subtracted. In algebra we may not be able to simplify the expression so readily as we do with numbers, and in that case *we must change the signs of all the numbers within a bracket which is preceded by the minus sign.*

Exercise 57

Simplify the following :—

1. $(3a-3b)+(2a-2b)-(4a-4b)$.
2. $7x-(8x+3x)+4x$.
3. $2a-\{5a+(3a-a)-(2a-5a)\}$.
4. $[a-\{a-(a-a)-a\}]$.

5. $3m-\{2n-(3p-2m)-3n\}-\overline{3p+3n}$.
6. $[4r-4s+\{2r-\overline{3r-2s}\}-2s+4r-s]$.
7. $\{2x-2y-(2x-\overline{2y-3x})+3x\}$.
8. $-[a-(b-2a)-\{2b-3a-\overline{3b-4a}\}-4b-5a]$.
9. $[l+m-\{n+l-(m+n-l)+m\}-n]$.
10. $\{2k-(2l-2k-\overline{2k-2l}-2l)\}$.

EXERCISE 58

1. Write the number which is one more than x and the number which is one less than x.

2. Write the next even number greater than $2x$, and the next even number less than $2x$.

3. How do you know that $2x$ is an even number ?

4. How many must be added to x to make it into y ?

5. If x pennies are placed in a box daily for y days, how many pounds may be then obtained for them ?

6. Find the cost of x eggs at m for threepence.

7. What must be added to $x+y+z$ to make the answer 0 ?

8. Write down the number of strides a man takes in walking a mile if each stride is b ft. long.

9. What must be added to $x+y$ to make it into 20 ?

10. What must be subtracted from $x+5$ to make it into 3 ?

11. If a grocer gains $\frac{1}{2}$d. on every egg he sells, how many eggs must he sell to gain x shillings ?

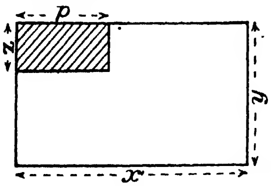


FIG. 6.

12. How many times can 2 be taken from r ?

13. If x be odd, write the two nearest even numbers.

14. Write the five consecutive whole numbers of which the middle one is x. Now find their sum.

15. Write the formula for the perimeter of the unshaded portion of fig. 6.

Multiplication

We know that multiplication is a contracted form of addition. Thus :

$$3 \text{ times } +4 = +4+4+4 = +12$$
$$3 \text{ times } -4 = -4-4-4 = -12.$$

We know also that $-3-3-3-3=-12$. $\therefore$ 3 times $-4=4$ times -3. Now consider the following operations:—

3 times $+4=+12$	3 times $-1=-3$
3 ,, $+3=+9$	3 ,, $-2=-6$
3 ,, $+2=+6$	3 ,, $-3=-9$
3 ,, $+1=+3$	3 ,, $-4=-12$
3 ,, $0=0$	

and so on.

Since the last result could be obtained by multiplying $+4$ by -3, we can build up another series.

-3 times $+4=-12$	-3 times $-1=3$
-3 ,, $+3=-9$	-3 ,, $-2=6$
-3 ,, $+2=-6$	-3 ,, $-3=9$
-3 ,, $+1=-3$	-3 ,, $-4=12$
-3 ,, $0=0$	

and so on.

Selecting four typical results, we find:

$$(+3)\times(+2)=+6$$
$$(+3)\times(-2)=-6$$
$$(-3)\times(+2)=-6$$
$$(-3)\times(-2)=+6.$$

This illustrates the **Rule of Signs**, namely : **the product of two terms with like signs is positive, and the product of two terms with unlike signs is negative.**

The Rule of Signs expressed algebraically is

$$(+\mathbf{a})\times(+\mathbf{b})=(+\mathbf{ab})$$
$$(+\mathbf{a})\times(-\mathbf{b})=(-\mathbf{ab})$$
$$(-\mathbf{a})\times(+\mathbf{b})=(-\mathbf{ab})$$
$$(-\mathbf{a})\times(-\mathbf{b})=(+\mathbf{ab}).$$

We know that $a^3=a\times a\times a$ and that $a^2=a\times a$.

$\therefore$ $a^3\times a^2=a\times a\times a\times a\times a=a^5$. Notice that this result is the product of 5 factors, each having the index 1. The index of the result is $1+1+1+1+1=5$. This is obtained more easily if we add the indices of the original like factors,

$$a^3\times a^2=a^{3+2}=a^5.$$

Similarly

$$x\times x^2\times x^3=x^{1+2+3}$$
$$5x^2\times 7x^3=5\times x^2\times 7\times x^3=5\times 7\times x^2\times x^3=35x^5$$
$$-4a^2b\times 3a^2b^2=-4\times 3\times a^2\times a^2\times b\times b^2=-12a^4b^3.$$

Examine fig. 7. The total area of the rectangle is equal to the sum of the areas of its parts.

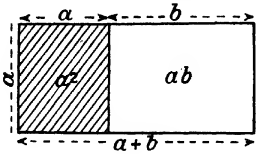


FIG. 7.

Now Area of any rectangle = BH.

$\therefore$ Area of shaded portion $= a \times a = a^2$.

And Area of unshaded portion $= b \times a = ab$.

And Area of whole rectangle $= a \times (a+b)$.

Now Area of whole rectangle = Shaded area + Unshaded area.

$$\therefore a(a+b) = a^2 + ab.$$

EXERCISE 59

Multiply, arranging each result in alphabetical order :—

1. a by b.
2. $3a$ by $4x$.
3. $5r$ by $3t$.
4. $4ab$ by $2a$.
5. x^2 by $-x$.
6. $-x^3$ by x^4.
7. $4a^2b$ by $3a^3$.
8. a^8 by $-a$.
9. $-a^3$ by $-a^2$.
10. $5mn^2$ by $3m^2n$.
11. a^2b^2 by $-a^2b^3$.
12. $9r^2t$ by $4r^2t^2$.
13. $-8abc$ by $-5rs^2$.
14. $7x^2y$ by $6x^3y^2$ by x.
15. $6ab^2c^2$ by xy by $-a^2$.
16. $2xyz$ by $9x^2y$.
17. $4x^2y$ by $4x^2y^3$.
18. $3ab$ by a^2 by $-2b^3$.
19. $5ar^3$ by $-r^2$ by $4ar^2$.
20. at^3 by $-2a^3$ by $-t^2$.

EXERCISE 60

1. Find the total length of x pens placed end to end, if each holder is $2a$ ins. $+b$ ins., and the nib protrudes a ins.

2. Fig. 8 gives the dimensions of a block of stone. How far will $x+30$ such blocks extend (i) if placed end to end? (ii) if placed side by side?

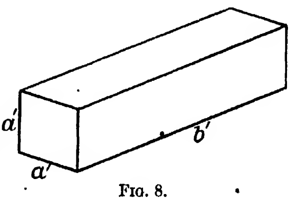


FIG. 8.

3. A sack of corn weighs y lbs. If x lbs. of corn are added and p lbs. are removed $3x$ times each, what weight remains in the sack?

4. A man pays m pence per mile on a railway journey. If the man travels x miles, $(y+2)$ miles, and ms miles, what will he pay as his fare?

5. If a box holds p pounds of soap, what must be paid for $(x-y)$ boxes if the price is p pence per pound ?

6. Between two towns A and B there are 576 telegraph poles. If the distance between the poles is 46 yds., find the distance from A to B in yds.

7. Write the formula for the area of the two walls shown in fig. 9.

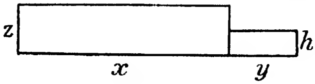


FIG. 9.

8. Write the formula for the area of the unshaded portion of fig. 10.

9. Write the formula for the area of the picture frame in fig. 11.

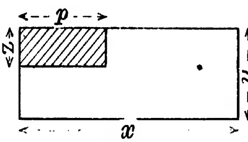


FIG. 10.

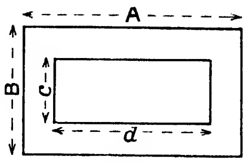


FIG. 11.

10. Find the average length of three lines which are respectively a ins., b ins., and c ins. long.

11. Find the volume of a cube of which the side is p ins. in length.

12. Find the area of blotting paper visible in a blotting pad of the given dimensions (fig. 12).

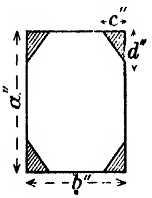


FIG. 12.

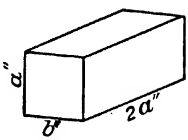


FIG. 13.

13. y blocks, each of the given dimensions (fig. 13), are just held in a box. Express the capacity of the box.

14. Give an expression for the area of the rectangle (i) in square inches, (ii) in square feet (fig. 14).

15. (*a*) Write an expression for the area of the piece of tin plate shown in fig. 15.

(*b*) If $b=2a$, state the answer in terms of a.

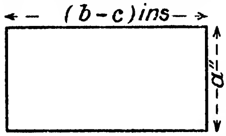


FIG. 14.

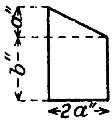


FIG. 15.

EXERCISE 61

Multiply:

1. $x+y-z$ by a.
2. $x-2y-z$ by $2b$.
3. a^3-3a^2+5 by $2c$.
4. $4a^2-3ab$ by $-5a$.
5. $x^2+2xy+y^2$ by xy.
6. $x^2-2xy+y^2$ by $-xy$.
7. a^3+3a^2b-4 by ab.
8. a^2c-3c^2 by $2c$.
9. $ab+b^2-b$ by $3b$.
10. $lm-n^3+5b$ by -5.
11. $a^2+2ab+b^2$ by $-ab$.
12. m^2-n^2 by $2mn$.
13. x^2-5y-z^3 by $+3$.
14. y^3+x^2y by $-4y$.
15. l^2-2mn by $2l$.
16. $5y-5y^2$ by $5y$.
17. $-3x^3-x^2y$ by $-5x$.
18. $2a^2-3a^3$ by $6a$.
19. $r^2-2rs+v$ by $2r$.
20. b^2-b-3 by $-2b$.

Division

We have seen that an algebraic fraction, such as $\frac{8xy^2}{24xy}$, can be cancelled. Thus, $\frac{8xy^2}{24xy}=\frac{y}{3}$.

We can perform simple algebraic disivions in the same way. Thus:

$$8x\div 2=\frac{2\times 4x}{2}=4x$$

$$5x^2\div x=\frac{5\times x\times x}{x}=5x$$

$$25a^3b^2\div 5ab=5a^{3-1}b^{2-1}=5a^2b.$$

EXERCISE 62

Divide:

1. a^2 by a.
2. b^2y by b.
3. $2a^2x$ by $2ax$.
4. x^9 by x^2.

5. $3abc^3$ by ab.
6. xyz by xy.
7. $7m^3n$ by mn.
8. rs^3 by rs^2.
9. $9d^3f$ by $3df$.
10. xy^3z by xyz.
11. $4a^3b^2$ by $2ab$.
12. lm^3n^3 by lmn.
13. $5p^3q^4$ by pq.
14. $x^3y^3z^3$ by x^2y.
15. $6xyz^5$ by $3z^4$.
16. a^3b^2 by ab^2.
17. $8x^3yz$ by $4yz$.
18. x^5m by xm.
19. yz^5 by y.
20. $10r^2l$ by $5r^2l$.

Using the typical cases on p. 63 and rearranging them, we obtain:

$$+6 \div +3 = +2$$
$$-6 \div +3 = -2$$
$$+6 \div -3 = -2$$
$$-6 \div -3 = +2.$$

And this gives the same Rule of Signs for Division as we obtained for Multiplication.

$$+\mathbf{x}^2 \div +\mathbf{x} = +\mathbf{x}$$
$$-\mathbf{x}^2 \div +\mathbf{x} = -\mathbf{x}$$
$$+\mathbf{x}^2 \div -\mathbf{x} = -\mathbf{x}$$
$$-\mathbf{x}^2 \div -\mathbf{x} = +\mathbf{x}.$$

Expressed in words, it becomes:

The quotient is positive when the dividend and the divisor have the same sign.

The quotient is negative when the dividend and the divisor have opposite signs.

EXERCISE 63

Divide:

1. a^5 by a^2.
2. x^8 by x^3.
3. a^2y^4 by ay.
4. x^5y^5 by x^2y.
5. $28a^2b^2c^2$ by $7ab$.
6. $35m^2n^3p$ by $5mnp$.
7. $4r^2s^3$ by $2rs$.
8. $3mnp^3$ by mn.
9. $4x^2y$ by $-2y$.
10. $-5x^2y^3$ by xy^2.
11. $8abc^2$ by $2c^2$.
12. $-4x^3y^3$ by $-2x$.

We may sometimes wish to express, by using brackets, terms which contain a common factor.

Example (i) $12a-6b+9c$
$=3(4a-2b+3c)$.

Example (ii) $5x+10y-8a-8b-6m+24n$
$=5(x+2y)-8(a+b)-6(m-4n)$.

Note the effect of the minus sign before a bracket.

Exercise 64

Rearrange in brackets :

1. $7a+14b+21c$.
2. $5ab-15bc+20abc$.
3. $7ax-14ac+5am-10an$.
4. $3b-7a+6c$.
5. $-3ax-6mx$.
6. $6bc-3by-2mn-6mp$.
7. $5xy-5xyz-15xz$.
8. $ar^2+xr+7br^2-5lr$.
9. $16ay-5ax+24az+15aw$.
10. $4a+4b-2d+18a+9b-9c$.

Simple Equations

A man sets out from home and after a journey of 2 miles takes the train and travels x miles. We may express the total distance he travels as $(x+2)$ miles.

If the value of x is fixed, then the value of $(x+2)$ is also fixed. Thus:

Condition, *i.e.* value we have fixed for x.	Result, *i.e.* value the expression must then have.
When $x=2$	$x+2=4$
$x=1$	$x+2=3$
$x=0$	$x+2=2$

The equation $x+2=4$ is only true if $x=2$
$x+2=3$,, ,, $x=1$
and so on.

If we say $4x=24$, it follows that x has only one value, namely, $\frac{24}{4}=6$, and we may check this solution, for $x=6$ is the only condition which satisfies the equation.

If in any equation
(i) We add an equal amount to each side, the sum totals on each side are still equal :
(ii) We subtract an equal amount from each side, the remainders on each side are still equal :
(iii) We multiply each side by an equal amount, the products of each side are still equal :
(iv) We divide each side by an equal amount, the quotients of each side are still equal.

We may illustrate the above truths by comparing an equation with a balance.

FIG. 16.

If $x = 20$ lbs. and we
(i) add equal amounts to each side ;
or (ii) subtract equal amounts from each side ;
or (iii) multiply each side by equal amounts ;
or (iv) divide each side by equal amounts ;
the balance remains in equilibrium.

Consider the equation $7x = 3x + 28$.

Subtracting $3x$ from each side, we get

$$7x - 3x = 3x - 3x + 28.$$
$$\therefore\ 7x - 3x = 28,$$
and
$$4x = 28.$$
$$\therefore\ x = 7.$$

Note.—Subtracting $3x$ from each side of the equation is equivalent to transferring the $3x$ from the right-hand side to the left-hand side and changing its sign.

Example.—Find the value of x *when*

$$\frac{x}{3} + \frac{x}{4} - \frac{x}{5} = 69.$$

Or, stated as a problem, $\frac{1}{3}$ *of a number* + $\frac{1}{4}$ *of a number* − $\frac{1}{5}$ *of a number* = 69. *Find the number.*

The first step is to get rid of the fractional quantities. This can be done by multiplying all the terms on *both* sides by a common denominator of the fractional quantities. The least common multiple of the denominators is 60.

Multiplying all the terms by 60, we obtain :

$$\frac{60x}{3} + \frac{60x}{4} - \frac{60x}{5} = 60 \times 69$$
$$\therefore\ 20x + 15x - 12x = 60 \times 69$$
$$35x - 12x = 60 \times 69$$
$$23x = 60 \times 69$$
$$x = \underline{180.}$$

Hints when working equations :—

1. When transferring one term to the other side of the equation be sure to change the sign.

2. Treat both sides of the equation in the same way, *i.e.* if the whole of one side is multiplied or divided by a quantity, the same process must be performed on the other side.

3. Aim at bringing the unknown quantity or unknown quantities to the left-hand side of the equation.

4. Check the solution by substitution in the original equation.

Exercise 65

Find the value of the x in the following equations :—

1. $5x-3=22.$	7. $14x-6=7x+22.$
2. $7x+4=25.$	8. $\frac{x+8}{9}=8-3.$
3. $2x+19+32=71.$	9. $20x-5=4x-105.$
4. $15x=75.$	10. $1+x=\frac{x}{2}.$
5. $\frac{x}{3}=14-5.$	11. $7x-5=3x+15.$
6. $5x-8=3x+16.$	12. $\frac{x}{4}=\frac{1}{2}.$

Exercise 66

1. Divide 28 into two parts, so that 3 times one part may be equal to 8 times the other part.

2. Find a number such that if 18 be added to it three times the sum will be 81.

3. To the half of a certain number I add 24, and the result is 32. What is the number ?

4. The sum of two numbers is 13. If three times the smaller number is added to four times the greater, we find the sum is 46. What are the numbers ?

5. Find two numbers whose sum is 100 and difference is 6.

6. Divide 70 into two parts, so that twice one part is five times the other part.

7. Divide 40 into two parts, so that when three times the greater part is added to four times the other part the sum is 139.

8. Father is 34 years old and his son is 7. In how many years will the father's age be just twice that of his son ?

9. A bill of £60 was paid in crowns and florins, and 40 more florins than crowns were used. How many of each coin were paid ?

10. A certain number of shillings, together with twice that number of half-crowns and three times the same number of

crowns, make £105. Find how many coins there are of each kind.

11. The half, together with the third and the fourth of a certain number, is 156. Find the number.

12. What three consecutive numbers add up to 54 ? (Let x be the middle number.)

13. Which five consecutive numbers add up to 105 ? (Let x be the middle number.)

14. What is the number whose eighth part exceeds its ninth part by 10 ?

15. Divide £100 among A, B, and C, so that A may have £20 more than B, and B may have £10 more than C.

16. Find a number such that the sum of its third and fifth parts exceeds the half of it by 17.

17. A man is now twice as old as his son ; eighteen years ago he was four times as old. What is the present age of the man ?

18. I bought a number of walnuts at 5 for a penny, and three-quarters of that number at 3 for a penny. I sold all of them at 25 for 8d., and gained 11d. How many walnuts did I buy ?

19. A room has a width two-thirds of its length. If the room were 8 ft. shorter in length and 8 ft. longer in width it would be a square. Find its dimensions.

20. X and Y start for a holiday, and X has £5 more money than Y. When they return X has spent half of his money and Y three-quarters of his, and now they have together £25. Find with how much money each started.

Exercise 67

1. Four times a certain number decreased by 6 gives 302. Find the number.

2. A girl is now half as old as her mother, but 12 years ago the mother was three times as old as the girl. Find the present ages of mother and daughter.

3. Seventy-eight yds. of string are cut into two pieces, so that $\frac{1}{9}$ of one piece together with $\frac{1}{8}$ of the other piece extend 9 yds. Find the length of each part.

4. A man's safe contains £9. In the safe are half-crowns and sixpences only. If he has four times as many sixpences as half-crowns, how many has he of each ?

5. There were $(a+b)$ children at school ; $\frac{1}{2}$ of them were

boys and $3b$ were girls in the mixed department. How many infants were there in the school ?

6. Find a number such that twice its third part is less than the number itself by 7.

7. One-quarter of a certain number is greater than one-fifth of the next consecutive number by one. Find the numbers.

8. In a game of cricket A makes $\frac{1}{4}$ of the runs, B makes 20 runs, and C makes the rest. If C scores 14 more runs than A, find the total score.

9. How much must be added to $m-n$ to make it equal to m^2-n ?

10. One hundred and twenty people attend a concert; some pay a shilling for their seats, and the rest pay sixpence each. If the total money drawn is £5, 7s. 6d., how many paid a shilling ?

11. A lady buys 7 lbs. of nuts at x pence per lb., and 4 lbs. of apples at $2x$ pence per lb. She had $7\frac{1}{2}$d. change out of 5s. Find the price per lb. of the nuts.

12. After buying 8 books at x shillings a copy, I have 8s. change out of £1, 10s. Find the price of each book.

13. A boy bought 2 rabbits and 6 guinea-pigs for 11s. 6d. If a rabbit cost 1s. 1d. more than a guinea-pig, what was the cost of each ?

14. What value of x will make $\frac{1}{3}(x-2)+\frac{3}{4}(x-4)$ equal to $\frac{8}{3}(x-5)$?

15. A's age is equal to the sum of the ages of B and C. Ten years ago A was twice as old as B. Show that ten years hence A will be twice as old as C.

16. A boy has to solve the equation $5x+27=92-8x$. He begins thus : $5x+8x=92-27$. State exactly what he has done to change the equation to this form.

17. After A has received £10 from B he has £6 more than B has then. Together they have £40. How much had each at first ?

18. AB (fig. 17) represents a running track, and a flag is placed at F. A man running from A to F runs 20 yds. further than a man running from F to B. How far is F from B if the track is 56 yds. long ?

FIG. 17.

19. An iceberg is observed floating towards a ship. The submerged part is 1 ft. and nine times longer than the exposed

part. If the total height of the iceberg is 407 yds., find the height of the exposed part.

20. A builder driving a peg into a wall finds he can drive it only so far that $\frac{1}{5}$ of the projecting part equals $\frac{1}{3}$ of the rest. If the peg is 6 ins. long, find the length projecting.

EXERCISE 68

1. During a football season a team won x matches, lost y matches, and drew z matches. Two points are awarded for winning, one point for a draw, and none for losing. Write down expressions giving :

(i) The total number of matches played.
(ii) The total number of points gained during the season.

2. Find the value of x in each of the following :—

(i) $7x-5=3x+15$, (ii) $\frac{x}{4}=\frac{1}{3}$, (iii) $3(x-4)-2(x+2)=2x-4$.

3. Given that $A=x+4$ and $B=2x-4$:

(i) Find the value of $2A-B$; (ii) What value of x will make A equal to 2B ?

4. Solve the equation $\frac{3}{4}(x+8)=\frac{2}{3}(6-x)+19$.

5. A square has a perimeter of $10x$ ft. What will be the length of one side in inches ?

6. A train travels at a speed of $20x$ miles per hour for $\frac{y}{5}$ hrs. How far will the train have travelled in miles ?

7. Solve the equation $4-2x=7-3(x+2)$.

8. If $d=\cdot 75$, and $p=\cdot 1$, find the value of D in the equation $D=d-1{\cdot}28\,p$.

9. If $C=\frac{E}{R}$, calculate E when $C=1{\cdot}2$ and $R=1{\cdot}1$.

10. A line has a length of x cm. What is its length in inches (2·5 cm.=1 in.) ? If a square be drawn on this line, what is its perimeter in inches ?

11. If $H=\frac{alnp}{33{,}000}$, arrange the formula so as to give l in terms of H, p, a, and n.

12. A tank contains $(a-b)\ (b-c)\ (d-c)$ cu. ft. of water. Given that $a=6$ ft., $b=4$ ft., $c=2$ ft., $d=8$ ft., find the volume of water in the tank.

13. One rectangle has a length of $3x$ ins. and a breadth of 12 ins. Another rectangle has a length of $2x$ ins. and a breadth of 18 ins. Write expressions for the perimeter of each rectangle. If the two perimeters are equal, find the value of x.

14. Which is the greater, $a-6$ or $a-8$? If $a-6=4$, find the value of $a-8$.

15. A train travels x miles in y hrs. What is its average rate in feet per second ?

16. A man walks for t hrs. at m miles per hour. If he returns the same way at 3 miles per hour, resting for r minutes, how many hours will the journey back take him ?

17. A rectangular sheet of glass is a ft. long, and its breadth is b ft. less than its length. What is its perimeter ?

18. When $A=\frac{BH}{2}$, find the value of H when $A=7{\cdot}4$ and $B={\cdot}37$. Also, find the value of B when $H=3{\cdot}09$ and $A=309$.

19. Express by brackets :

(i) The sum of r, s, and t, multiplied by a.

(ii) x times the difference between y and z, when y is greater than z.

(iii) The product of $x+y$ and the sum of a and b.

20. The head of a fish is $\frac{1}{3}$ the length of the body ; the tail is $\frac{1}{4}$ the length of the head and body together ; the body is 6 ins. longer than the tail. What is the length of the fish ?

CHARTS AND GRAPHS

Charts

Using squared paper, we are readily able to show the relative heights of the pupils in a class. Thus, assuming that one division of the squared paper represents 2 inches, fig. 18 shows the comparative heights of pupils : 4 ft. 0 in., 4 ft. 2 ins., 4 ft. 4 ins., 4 ft. 8 ins., 4 ft. 10 ins., 5 ft. 0 in., 5 ft. 4 ins.

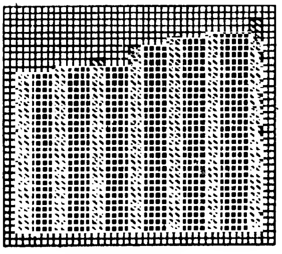


Fig. 18.

Again using squared paper, we may show the approximate relative standard values of a franc ($9\frac{1}{2}$d.), a mark ($11\frac{3}{4}$d.), a rupee (1s. 4d.), a dollar (4s.), and a shilling. Assuming that

one division of the squared paper represents a penny, we obtain fig. 19.

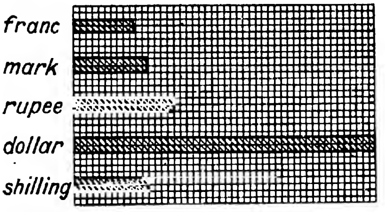


FIG. 19.

The object of these charts is to make the comparison more obvious.

EXERCISE 69

Represent the following statements by charts on squared paper :—

1. The population of the British Empire in round numbers is : Europe, 45 m. (m.=million) ; Asia, 330 m. ; Africa, 50 m. ; America, 11 m. ; Australia, $7\frac{1}{2}$ m.

2. A threepenny piece weighs 1·4 grams ; a sixpenny piece 2·8 grams ; a two-shilling piece 11·2 grams. Show in a similar way the value of a silver coin weighing 10·5 grams.

3. The weights in pounds of 1 cu. ft. of the following substances are : wax, 60·5 lbs. ; bronze, 534 lbs. ; charcoal, 22·4 lbs. ; aluminium, 160 lbs. ; brass, 524 lbs. ; copper, 548 lbs.

4. The areas of the following countries are : Belgium, 11,400 sq. miles ; Denmark, 15,500 sq. miles ; Netherlands, 12,800 sq. miles ; Switzerland, 16,000 sq. miles.

5. The temperatures recorded by an observer in investigating the melting-point of wax were : 32° C., 36° C., 41° C., 46° C., 52° C., 51° C., 51° C., 51° C., 57° C., 64° C.

6. The constitution of a certain crystal is 40 per cent. copper, 20 per cent. sulphur, 40 per cent. oxygen.

7. The areas of the following continents in millions of square miles are : Europe, 3·8 ; Asia, 17 ; Africa, 11·5 ; North America, 8 ; South America, 6·8.

8. The history of England since 1066 is divided into periods as follows : 1066–1154, Norman ; 1154–1485, Plantagenet ;

1485–1603, Tudor; 1603–1714, Stuart; 1714–1910, Hanoverian; 1910– , Windsor.

9. The sources of British supply of apples are: United States, 50 per cent.; Canada, 34 per cent.; Australia, 7 per cent.; Portugal, 2 per cent.; Belgium, 2 per cent.; other sources, 5 per cent.

10. The annual trade of the following countries is:

United Kingdom . .	1200 million pounds.
British Possessions .	803 ,, ,,
United States . .	863 ,, ,,
France	690 ,, ,,
Germany . . .	690 ,, ,,

SECTION IV

GEOMETRY AND MENSURATION

Length

Solids, such as cubes, prisms, cylinders, etc., are known as *regular* solids. They have length, breadth, and thickness. They are enclosed by level surfaces which have two dimensions, namely, length and breadth. These surfaces or planes are enclosed by lines which have one dimension only, namely, length. Of course, when we draw a line we must show some breadth, if it is to be seen at all, but we are really indicating direction only—the direction in which a point is supposed to move. When two lines cross or intersect one another they cross at a point. This point has position only--no length, no breadth, no thickness. We may say then :

A Point has no magnitude ; it indicates position only.

Length is the distance between two given points.

A Line is measured in length ; its width is not considered.

A Straight Line is the shortest distance between two points, *i.e.* between two definite positions.

Perimeter is the boundary of any plane figure.

Measurement

EXERCISE 70

1. Draw lines : $2\frac{1}{4}$ ins., $1\frac{3}{4}$ ins., $3\frac{3}{8}$ ins., 1·6 ins., 1·9 ins., 2·1 ins., 5·8 cm., 6·7 cm., 9·2 cm.

2. Draw lines as nearly as possible : 1·25 ins., 1·35 ins., 2·15 ins., 1·85 ins., 3·05 ins.

3. Measure the length and breadth of this page : (i) in inches and decimals of an inch, (ii) in cm. and mm.

4. Draw lines 1 in., 2 ins., 3 ins., 4 ins. long. Using the ruler, find the length of each of these four lines in cm. and mm. Find the total of these lengths in cm. and mm. This total

=1 in.+2 ins.+3 ins.+4 ins.=10 ins. What is the length of 1 in. in centimetres ?

5. Draw lines 5 cm., 10 cm., 25 cm. in length. Using the ruler, find the length of each of these three lines in inches and decimals of an inch. Find the totals of these lengths. This total=5 cm.+10 cm.+25 cm.=40 cm. What is the length of 1 cm. in inches ?

6. Draw a line 10·5 cm. long on your paper. What is the length of this line in millimetres and also in inches ? Now, from the length of this line in inches and in centimetres, find how many centimetres are contained in 1 in.

7. A certain type of areoplane engine has cylinders, each of which is $5\frac{1}{2}$ ins. in diameter. Find the diameter of one of the cylinders in millimetres and then in centimetres and decimals of a centimetre.

8. Draw two lines (*a*) $\frac{3}{8}$ of 4 ins. long, (*b*) 4 times $\frac{3}{8}$ in. long. Compare their lengths.

9. The lengths of the different parts of a steel spindle are given in inches in fig. 20; what is the total length of the spindle in inches and also in feet ? Find the average in inches of the lengths given.

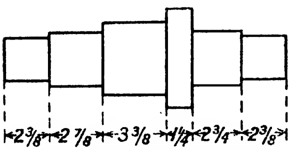


Fig. 20.

10. A train 400 ft. long, travelling at 40 miles per hour, passes another train 480 ft. long travelling at 15 miles per hour. How long do they take to clear each other when travelling (*a*) in opposite directions, (*b*) in the same direction ?

Scales

As it is often inconvenient to make the drawings of objects full size, we make drawings *to scale*. This means that every line in the drawing is a definite fraction of the distance it represents. If 1 ft. is represented by 1 in., the scale is 1 in. to 1 ft., or the scale is $\frac{1}{12}$.

Fig. 21 shows a scale of 1 in. to 1 ft.

The line AB represents a distance of 3 ft. 8 ins., but its actual length is $3\frac{2}{3}$ ins.

The fraction $\dfrac{\text{Number of units in a line}}{\text{Number of units the line represents}}$ is called *the*

Representative Fraction or *the Equivalent Fraction* for the scale. In the above example the Representative Fraction is $\frac{1}{12}$.

A scale of $1\frac{1}{2}$ in. to 1 ft. has a Representative Fraction $\frac{1\frac{1}{2}}{12}=\frac{1}{8}$.
" " 5 cm. to 1 m. " " " $\frac{5}{100}=\frac{1}{20}$.

Notes.—1. The dimensions on the scale show the real measurements represented.
2. The *open parts* of the scale given above are the parts from 0′ to 3′.
3. The *divided part* of the scale is the part from 0′ to 12″.

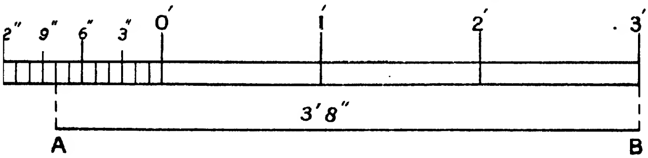


Fig. 21.

Exercise 71

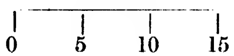
1. Construct a scale 2 ins. to 1 ft., show 3 ft., and draw lines representing 2 ft. 7 ins., 1 ft. 8 ins., and 2 ft. 3 ins.

2. Construct a scale 1 in. to represent 1 mile. From this scale draw lines representing 1 mile 5 fur., 2 miles 3 fur., 3 miles 7 fur.

3. Find the representative fractions of the following scales: 1 in.=1 mile, 1 in.=1 yd., $\frac{1}{2}$ in.=2 ft., 4 ins.=5 yds.

4. This is a scale of miles given on a map of Lancashire:

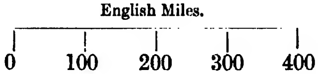
Scale of Miles.



Find (i) What the scale is, (ii) the representative fraction, (iii) the real distance between Manchester and Cark, which are $5\frac{1}{2}$ ins. apart on the map.

5. Here is a scale of miles given on a map of Europe:

English Miles.



Find (i) What the scale is, (ii) the representative fraction, (iii) the real distance from Archangel to Baku, which, when measured on the map, is 5·1 ins.

6. (*a*) Fig. 22 illustrates a portion of an inch scale. What is the value of the smallest sub-divisions of the scale ?

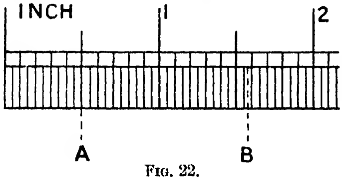


Fig. 22.

(*b*) Express the distance, *as shown in the figure*, between the two lines A, B, in inches, to *two* decimal places.

(*c*) The distance between the lines A, B, is also equal to 2·72 cm. Hence calculate the number of millimetres in 10 ins.

7. A town is 3·3 miles E. and 4·2 miles S. of a definite landmark. (1) Draw a scale 1 in. to 1 mile, showing tenths of a mile. (2) Make a drawing to the foregoing scale and show the relative positions of the town and the landmark. (3) Find the shortest distance between the two positions by means of your drawing and scale.

8. The line AB (fig. 23) represents an aerial for a wireless

Fig. 23.

installation to a scale of $\frac{1}{16}$ in. represents 1 ft. Find its length in feet and inches.

9. Show on squared paper a point Q, 2·6 miles to the west and 3·5 miles to the south of a fixed point O. Measure the distance OQ. Work to a scale of 1 in.=1 mile.

10. Draw a line to represent the shortest distance from London to Moscow (27,000 km.).

Scale : 0 2250 km.

Circumference of a Circle

We have already said that when a point moves consistently in one direction, say to the North or the East, it traces a line. Consider another path in which a point may move : Knot a piece of thread to a pin and stick the pin upright and firmly into a sheet of paper ; fasten the loose end of the thread to a pencil. Now, with the pencil, trace the line described by a point

which always maintains the same distance from the pin-point. We have drawn the *circumference* of a circle.

Fig. 24 shows 32 points, each $\frac{1}{2}$ in. from the point O. Notice the shape of the figure produced by joining them in order. What would be the effect of increasing the number of points to 64; 100; 1000; an infinite number?

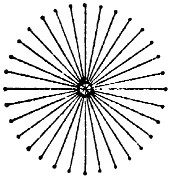


FIG. 24.

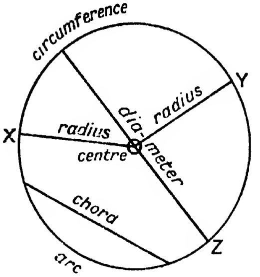


FIG. 25.

XYZ (fig. 25) is the *circumference* of a circle. All the points on it are exactly the same distance from its *centre* O. Thus OX=OY=OZ, etc.

The space within this boundary or *circumference* XYZ is the *circle*.

Any straight line from the centre O to the circumference is a radius of the circle (the plural of radius is *radii*).

Any straight line drawn from a point on the circumference through the centre to another point on the circumference is a *diameter* of the circle. It is evident that a diameter is made up of two radii which are in the same straight line.

An arc is any portion of the circumference of a circle.

A chord of a circle is a straight line joining any two points on the circumference.

Notes.

1. The diameters are the greatest chords that may be drawn in a circle.
2. Any chord not passing through the centre of the circle is less than the diameter.

EXERCISE 72

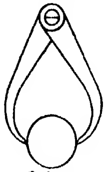
1. Measure with thread or with dividers the curve drawn below.

FIG. 26.

2. (*a*) Obtain a cylinder and then wrap thread round it an *exact* number of times. From the length of the thread

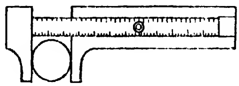
used, find the distance *once* round the cylinder (*i.e.* the circumference).

(*b*) Find the diameter of the cylinder by one of the methods shown below in figs. 27, 28, 29.



Calipers

Fig. 27.



Slide Calipers

Fig. 28.

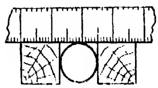


Fig. 29.

(*c*) Find the ratio of the circumference to the diameter, *i.e.* find the number of times that the length of the circumference contains the length of the diameter

Note.—This ratio is called π (pronounced *pie*).

3. Find the value of π again, using any circular object at hand, *e.g.* a coin, a cocoa-tin, a lid.

Set out your results like this :

	Circumference.	Diameter.	Value of π.
Coin . . .			
Cocoa-tin . .			
Lid . . .			

4. If C=circumference, d=diameter, r=radius, then we may say : $d=2r$, $r=\frac{d}{2}$, $\frac{C}{d}=\pi$, $C=\pi d$, $\frac{C}{2r}=\pi$, $C=2\pi r$.

(*a*) Express d in terms of (i) r, (ii) C and π.
(*b*) ,, r ,, ,, ,, (i) d, (ii) C and π.
(*c*) ,, C ,, ,, ,, (i) r and π, (ii) d and π.
(*d*) ,, π ,, ,, ,, (i) C and r, (ii) C and d.
(*e*) Find to two places of decimals the value of d when $\pi=3{\cdot}14$ and $C=5{\cdot}55$ ins.

(*f*) Find the value of r when $\pi=3{\cdot}14$ and $C=7\frac{1}{2}$ ins.
(*g*) ,, ,, ,, ,, C when $\pi=3{\cdot}14$ and $r=5\frac{3}{4}$ ins.

Note that the exact value of π cannot be determined. Sometimes it is taken as $3\frac{1}{7}$, sometimes as 3·14, sometimes 3·1416, according to the degree of accuracy desired.

5. When π is reckoned as 3·14, find the circumference of

(*a*) A wheel, if its diameter is 3·6 ins.
(*b*) A circular garden plot, if its diameter is 9·7 metres.
(*c*) The top of a circular lid, if its diameter is 9·3 cm.
(*d*) A circular racecourse, if its diameter is 50 yds. 2 ft.

6. When π is reckoned as $\frac{22}{7}$ or $3\frac{1}{7}$, find the diameter of

(*a*) A circular pond, if its circumference is 1 fur. (Answer in yards.)
(*b*) A cylindrical boiler, if its circumference is 10 yds. (Answer in yards.)
(*c*) A cylindrical gasholder, if its circumference is 50 metres. (Answer in metres.)

7. If the minute hand of a clock is 5·2 ins. long, find the distance its point travels in 45 mins. ($\pi=3{\cdot}14$.)

8. The wheels of a motor-car are all 30 ins. in diameter. When the car is being driven at 25 miles per hour, how many revolutions per minute are the wheels making? ($\pi=\frac{22}{7}$.)

9. A bicycle, having wheels 28 ins. in diameter, was being ridden along a road and made 500 revolutions per minute. Find the speed (*a*) in feet per minute, (*b*) in miles per hour. ($\pi=3\frac{1}{7}$.)

10. A wire rope is wound on to a circular drum at a constant radius of 3 ft. 6 ins. Find how many turns the drum must make to wind on 60 yds. of rope. ($\pi=\frac{22}{7}$.)

11. If a garden roller is 2 ft. 9 ins. in diameter, and it rolls 14 times in travelling from end to end of a lawn, how long is the lawn? ($\pi=3\frac{1}{7}$.)

12. In travelling round a circus 5 times a horse runs $235\frac{1}{2}$ yds. How far is the horse from the circus master standing in the centre of the circus? ($\pi=3{\cdot}14$.)

13. The minute hand of a clock is 12 ins. long, and the hour hand is 9 ins. long. How much further does the minute hand travel than the hour hand in 18 hrs.? ($\pi=3\frac{1}{7}$.)

14. What length of wire has been employed in constructing a framework as shown in fig. 30? ($\pi=3{\cdot}14$.)

15. The diameter of the inside edge of a wheel is $56x$ ins.

If the wheel has $4x$ spokes, find the distance between the ends of the spokes. ($\pi = \frac{22}{7}$.)

16. A pulley is 2 ft. in diameter. What length of its driving belt passes over it in each minute if it makes one revolution per second? ($\pi = \frac{22}{7}$.)

17. Supposing the earth to be a perfect sphere of radius 4000 miles, find in miles the distance between each meridian of longitude at the equator. ($\pi = \frac{22}{7}$.)

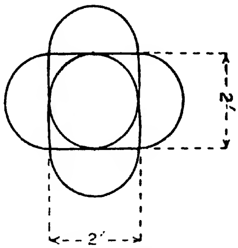


FIG. 30.

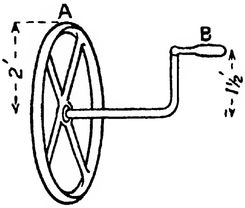


FIG. 31.

18. A small drop of ink is placed on the edge of a halfpenny (diameter = 1 in.), and the coin rolls along a paper, leaving 3 ink marks behind it. What is the distance between the first and last mark? ($\pi = 3{\cdot}14$.)

19. A girl's hoop has an outside diameter of 3 ft. 6 ins. How far does it travel in making $18\frac{1}{2}$ complete turns? ($\pi = \frac{22}{7}$.)

20. A wheel is connected by a shaft to a handle as in fig. 31. How much farther does A travel than B when the handle makes 10 turns? ($\pi = 3{\cdot}14$.)

Angles

Keeping one arm of a pair of compasses in a fixed position, turn the other arm about the hinge. The rotating arm has formed an *angle*. There is an infinite number of positions in which the rotating arm may be placed, and in each position a different angle is traced out.

Whenever a line fixed at one extremity rotates, it traces out an angle. Notice the angles made by the bottom edge of a door when it is wide open, half open, ajar. We say the door makes an angle with its initial position.

Consider the line AB (fig. 32) as pivoted at A. It may move to the position AC and form the angle BAC (written $\angle$ BAC), or it may move to the position AD and form $\angle$ BAD.

Note.—$\angle$BAC+ $\angle$CAD= $\angle$BAD.

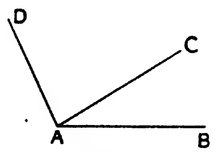


FIG. 32.

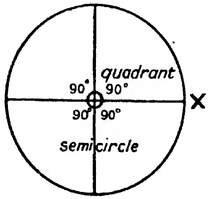


FIG. 33.

When a line OX (fig. 33), pivoted at O, makes one complete revolution, it is said to have moved through an angle of 360 degrees (written 360°), i.e. *a circle*; when through half a revolution, an angle of 180°, i.e. *a semicircle*; when through a quarter of a revolution, an angle of 90°, i.e. *a quadrant.* An angle of 90° is called a *right angle*; an angle of 180° *a straight angle.*

An Acute Angle is less than a right angle, *e.g.* $\angle$ AOC is an acute angle (fig. 34).

An Obtuse Angle is greater than a right angle, *e.g.* $\angle$ AOD is an obtuse angle (fig. 34).

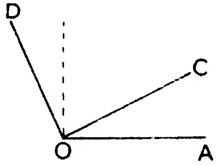


FIG. 34.

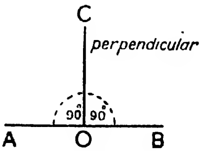


FIG. 35.

If one straight line stands on another straight line and makes the two adjacent angles equal to one another, each of these angles is a *right angle, e.g.* $\angle$'s AOC, COB are both right angles (fig. 35).

If one straight line cuts another straight line and makes four angles, each equal to one another, each of these angles is a *right angle, e.g.* $\angle$'s AOC, COB, BOD, DOA are all right angles (fig. 36).

Note, in fig. 36 AO is *perpendicular* to COD, and CO is perpendicular to AOB. What can be said of BO and DO ?

Note, in fig. 35 CO is perpendicular to AOB.

Consider $\angle$ AOB (fig. 37). It may be considered in two ways, viz. :

(*a*) It is the inclination of two lines AO and BO which meet at O.

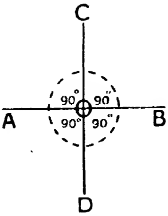


Fig. 36.

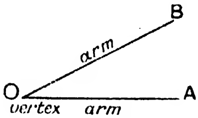


Fig. 37.

(*b*) It is the angle formed by the rotation of OA (fixed at O) from the position OA to the position shown by OB.

The two lines OA, OB are called the *arms of the angle.* The point O is the *vertex* or *angular point* of the angle AOB.

The *magnitude* of $\angle$ AOB depends upon the directions of the arms OA and OB, and not upon their length.

Exercise 73

1. Draw three circles of 2 ins. radius on gummed paper. Cut out the circles carefully and proceed with each as follows:—

1st. Draw a radius. Show by a dotted line and an arrow the path in which the radius must revolve to trace out an angle of 360°. Gum the circle in a notebook and label it 360°.

2nd. Fold and cut it along a diameter. Indicate the angle of 180°. Gum one semicircle in the book and label it. Fold, cut, indicate, gum, and label portions of the circle which are angles of 90° and 45°.

3rd. Set the compasses to a distance equal to the radius and mark off chords round the circumference. Notice how often this process may be carried out. Join the extremities of one of the chords to the centre of the circle. This makes an angle of 60°. Cut it out and indicate the rotation; gum and label it.

By folding and cutting bisect another angle of 60° cut from the same circle. Deal with this as before.

2. Examine a 60° and 45° set square. Learn the magnitude of each angle of each set square (fig. 38). Draw both set squares half size.

3. Arrange the set squares so as to make angles of 75°, 105°, 120°, 150°.

4. Examine your protractor and then use it to measure the

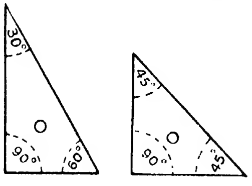


FIG. 38.

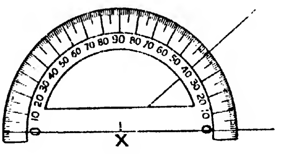


FIG. 39.

angles gummed in the notebook. In each case if X (see fig. 39) is placed at the vertex of the angle and the zero mark over one arm of the angle, then the graduation on the curved edge of the protractor gives the magnitude of the angle measured.

5. Through what angle does the minute hand of a clock move in $\frac{1}{4}$ hr., $\frac{1}{2}$ hr., $\frac{3}{4}$ hr., 5 mins., 10 mins., 55 mins., 50 mins., 20 mins., 40 mins., 25 mins., 35 mins. ?

6. What fraction of a right angle is 1°, 30°, 60°, 45° ?

7. What fraction of a circle is described by a line tracing out 1°, 15°, 75°, 270° ?

8. How many degrees are there in x right angles ?

9. How long does it take the minute hand of a watch to move through 60°, 90°, 240° ?

10. The pointer of a weather vane points North. What angle does it describe if it changes its direction (*a*) to the South ? (*b*) to the West ? (*c*) to the East ?

Useful Constructions

I. To bisect a straight line

A straight line may be bisected thus:

With centre A (fig. 40) and any radius greater than $\frac{1}{2}$ AB, describe two arcs, one above the line and the other below it. With centre B and the same radius as before, describe two arcs intersecting the first two arcs at C and D. Join CD, cutting the line AB at E. E is the mid-point of AB. Test with ruler.

II. To bisect an angle

From the vertex O (fig. 41) of the angle, describe an arc cutting the arms of the angle at A and B. With centre A and a convenient radius, describe an arc which lies within the angle. With centre B and the same radius, describe an arc to cut the first arc at C. Join CO. Then $\angle COA = \angle COB = \frac{1}{2}\angle AOB$. Test with protractor.

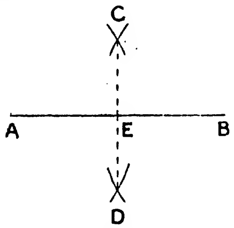


FIG. 40.

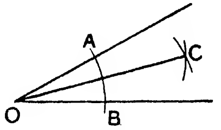


FIG. 41.

III. To draw a perpendicular to a given line

(i) *From a point in the line*

Let PQ (fig. 42) be the given line and O the given point in it. With centre O and a convenient radius cut off equal lengths on each side of O. Then OA=OB. With centres A and B draw arcs above the line as in the construction for bisecting a line given above. Let the arcs intersect at C. Join OC. OC is the required perpendicular. Test with protractor.

(ii) *From a point outside the line*

Let PQ (fig. 43) be the given line and O the given point outside it. With O as centre and a convenient radius, describe an arc cutting PQ at A and B.

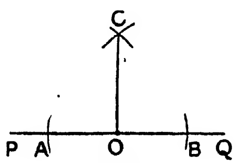


FIG. 42.

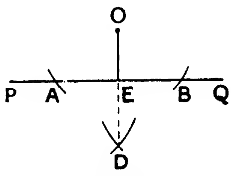


FIG. 43.

From A and B draw arcs below the line as in the construction for bisecting a line. Let the arcs intersect at D. Join OD, cutting PQ at E. OE is perpendicular to PQ.

IV. To copy a given angle

Let $\angle RPQ$ (fig. 44) be the given angle. It is required to copy the angle on the line OS (fig. 45), with the vertex at O.

With centre P and any convenient radius describe an arc to cut PR in M and PQ in N. With centre O and the same radius draw an arc cutting OS in B. Set the compasses to the distance NM, and with B as centre describe an arc cutting the other arc at A. Join OA and produce to T. Then $\angle TOS = \angle RPQ$. Test with protractor.

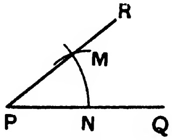


FIG. 44.

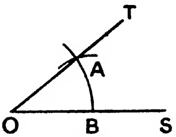


FIG. 45.

EXERCISE 74

1. Describe a circle, and on it show the points of the compass (see fig. 46).

2. Give, without using a protractor, the magnitude of the angle between N and E, N and S, E and SE, S and W, W and NW.

3. Construct geometrically (i) a right angle and bisect it; (ii) an angle of 60° and bisect it; (iii) an angle of 45° and bisect it (see p. 88).

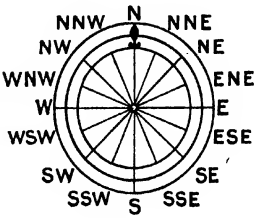


FIG. 46.

4. Construct by means of a protractor angles of 40°, 55°, 75°, 95°, 105°, 150°, 170°. Label them *acute* or *obtuse* in each case.

5. Show by drawings the angles made by the hands of a clock at 2 o'clock, 7 o'clock, 9 o'clock. (Note that there are two angles formed in each case.)

6. When $\angle A = 60°$, $\angle B = 45°$, $\angle C = 30°$, construct angles

$$= \frac{A+B}{2}, \frac{A+C}{2}, \frac{B+C}{2}.$$

7. From a point O draw 5 lines. Letter the angles so formed. Measure the magnitude of the angles. What is their sum ?

8. Draw two lines AB and CD intersecting at X. Name the angles so formed. Measure the angles. What do you observe about their sum ?

9. At P, a point in AB, draw $\angle OPB = 75°$. Without

measuring state the magnitude of ∠OPA. Test this by a protractor.

10. Copy the given ∠ABC (fig. 47). Produce CB to D and AB to E. Show that ∠CBE = ∠ABD, and that ∠EBD = ∠ABC.

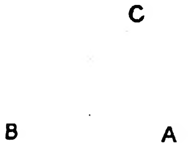


Fig. 47.

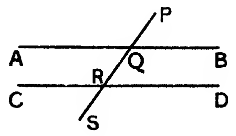


Fig. 48.

Parallel Straight Lines

Parallel lines are always the same perpendicular distance apart. Examples of parallel lines are the rails of a railway track, the edges of a book, etc. Parallel lines never intersect, however far they are produced in either direction.

The lines AB and CD (fig. 48) are parallel to one another. The line PQRS cuts the parallel lines in Q and R.

By using tracing paper show that:

(i) ∠AQP = ∠RQB; ∠PQB = ∠AQR; ∠CRS = ∠QRD; ∠QRC = ∠SRD. These are pairs of *opposite angles.*

(ii) ∠AQR = ∠QRD; ∠BQR = ∠QRC. These are pairs of *alternate angles.*

(iii) ∠PQA = ∠QRC; ∠AQR = ∠CRS; ∠PQB = ∠QRD; ∠BQR = ∠DRS. These are pairs of *corresponding angles.*

Exercise 75

1. Draw two parallel lines AB and CD. Draw a line PQRS at right angles to both AB and CD. Are the angles named above equal now?

2. Draw a straight line and keep the ruler along it. Place the edge of a set square against the ruler and draw a line along one of the other edges of the set square. Keep the ruler fixed, slide the set square along to another position, and draw a line as before. Remove the instruments. Mark the angles which are equal. What can be said about the lines?

3. Apply the fact learned in Q. 2 to construct rectangles 2 ins. × 4 ins., 3 ins. × $2\frac{1}{2}$ ins., 1·5 in. × 1·5 in., 3 cm. × 10 cm.

4. Draw $\angle ABC = 80°$, and mark a point O anywhere within the angle. Through O draw lines parallel to AB and CB and write down the sets of equal angles formed.

5. Draw a line XY $2\frac{1}{2}$ ins. long, and bisect it at O. Make $\angle AOX = 60°$, with OA $1\frac{1}{2}$ in. in length. Make $\angle OAB$ alternate and equal to $\angle AOX$. What can be said of AB and XY ?

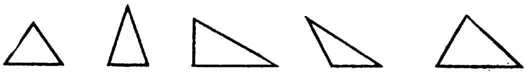
6. If railway lines are 4 ft. 3 ins. apart, make 3 drawings (scale $\frac{1}{2}$ in. = 1 ft.) to illustrate the crossing of two pairs of railway lines at angles of 65°, 80°, and 90° respectively.

7. Make an angle of 75°, and from a point in one arm draw a line parallel to the other arm. Measure and write the magnitude of the angles formed.

8. Draw a line XY 13 cm. in length, and at X construct $\angle YXZ$ 38°. At Y draw the $\angle XYM$ 142°. Prove that the lines XZ and YM are parallel.

The Triangle

A triangle is a figure bounded by three straight lines.



I II III IV V

FIG. 49.

Special Triangles.—If a triangle has

(i) Three equal sides and three equal angles, it is an *equilateral triangle* (see No. I, fig. 49).

(ii) Two equal sides and two equal angles, it is an *isosceles triangle* (see No. II, fig. 49).

(iii) All its sides and all its angles unequal, it is a *scalene triangle* (see Nos. III, IV, V, fig. 49).

(iv) One of its angles a right angle, it is a *right-angled triangle* (see No. III, fig. 49).

(v) One of its angles obtuse, it is an *obtuse-angled triangle* (see No. IV, fig. 49).

(vi) All its angles acute, it is an *acute-angled triangle* (see Nos. I, II, V, fig. 49).

EXERCISE 76

1. Construct an isosceles triangle on a base $1\frac{1}{2}$ in. long and having its equal sides each 2 ins. long.

2. Construct a scalene triangle having its sides $1\frac{1}{4}$ in., $1\frac{1}{2}$ in., and $1\frac{3}{4}$ in. respectively.

3. Find three points P, Q, and R, so that each point is 2 ins. from the other two.

4. Construct an equilateral triangle having sides of $1\frac{1}{2}$ in., and then construct an equilateral triangle on each side of it.

5. Draw a line AB $1\frac{1}{2}$ in. long, and on it construct two equilateral triangles ABC above the line and ABC′ below the line. Join CC′ to cut AB at O. Compare the construction with the bisection of a line on p. 87. Measure the lengths AO, BO.

6. Construct triangles to the following data and name each according to its properties :—

(i) ABC when AB=BC=AC=1·5 in.
(ii) OPQ ,, OP=3·2 ins., PQ=2 ins., OQ=2 ins.
(iii) RST ,, RS=2·5 ins., ST=2 ins., RT=1·5 in.
(iv) MNO ,, MN=3 ins., $\angle$MNO=120°, NO=$2\frac{1}{2}$ ins.

7. On a base of 2 ins. construct a triangle with its sides equal to its base. Measure each angle. Explain the statement, " An equilateral triangle is equiangular."

8. On each side of a line $2\frac{1}{2}$ ins. long construct any isosceles triangle. Join the opposite vertices and find where the line cuts the base.

We have seen that a triangle may be constructed if the positions of each of its three vertices are known. If we are told the length of one side of a triangle, the position of two vertices is fixed. The third vertex is fixed if we also know

(*a*) The lengths of the other two sides, or
(*b*) The magnitude of the angles at the base, or
(*c*) The magnitude of one base angle and the length of its other arm.

Exercise 77

1. Construct triangles to satisfy the following conditions :—

(i) ABC having AB=2 ins., BC=1·5 in., AC=1·8 in.
(ii) XYZ ,, XY=2 ins., $\angle$XYZ=45°, YZ=$1\frac{3}{4}$ in.
(iii) PQR ,, PQ=3 ins., $\angle$RQP=30°, $\angle$RPQ=47°.

2. Draw any triangle ABC and then construct a triangle DEF equal to it in all respects.

3. A straight road connects a post office in a town M with a post office 5 miles away in a town T. A third post office is

situated at P on a road inclined to the first road at an angle of 30° at M, and this post office is 7 miles from the office at M. Find from a drawing how far the post office at P is from the office at T. (Scale 1 in.=2 miles.)

4. From each end of a straight-sided fort two fixed guns are set at angles of 62° and 43° respectively to the side of the fort, which is 1 mile long. Show by a drawing the position of a tower which both guns could demolish.

5. Two stations, X and Y, are 8 miles apart. A train from X arrives at a station S after a journey of 6 miles in a straight line, and a train from Y arrives at S by a journey of 7 miles, also in a straight line. Indicate on a drawing the relative positions of XY and S.

6. An aeroplane is in a position above a straight road 4 miles in length. At each end of the road an observer finds that a line from himself to the aeroplane is inclined at an angle of 63° to the line of the road. Using a scale 1 inch=2 miles, make a drawing to show the relative positions of the aeroplane and the observers.

7. Construct an isosceles triangle having a base 2·1 ins. in length and the sum of its base angles 130°.

8. A man walking along a canal bank knows that he is 50 ft. away from a tree on the opposite bank. When he has walked 38 ft. the tree is 32 ft. away from him. Show the relative positions of the man and the tree when the observations were made. (Scale 1 in.=20 ft.)

Angles of a Triangle

On gummed paper draw any triangle and mark its angles as shown in fig. 50. Cut out the triangle, tear off its three corners,

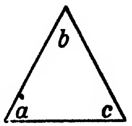


FIG. 50.

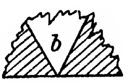


FIG. 51.

and gum them in the notebook as shown in fig. 51. The exterior arms form a straight line and, therefore, the sum of the angles of a triangle = a straight angle = 180° = 2 right angles.

Exercise 78

1. Two angles of some triangles are respectively (i) 56°, 74°; (ii) 28°, 126°; (iii) 45°, 45°; (iv) 60°, 75°; (v) 15°, 15°. Find in each case the magnitude of the remaining angle.

2. If one angle of an isosceles triangle is a right angle, which angle must it be? What can be said of the other two angles?

3. In fig. 52 $\angle QOP=90°$, $\angle QPO=38°$. What are the magnitudes of the angles *a, b, c, d* when OR is drawn perpendicular to PQ?

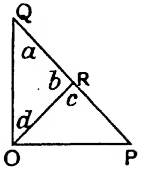


Fig. 52

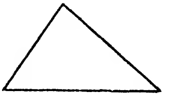


Fig. 53.

4. Draw a scalene triangle ABC and produce AB to D. Measure $\angle BAC$, $\angle ACB$, and $\angle CBD$. What do you observe?

5. On a base of 2 ins. construct a triangle having its base angles equal to those of the given triangle (fig. 53). Use a ruler and compasses only.

6. Construct a triangle XYZ from the following data: YZ=13·5 cm.; angle XYZ=37°; angle XZY=86°. Measure the angle YXZ and state its magnitude.

7. On a base 2·9 ins. long construct an isosceles triangle which has a vertical angle of 30°. Find the magnitude of its base angles.

8. CDE is a triangular field of which side DE is 120 yds. The boundary CD makes an angle of 72° with DE, and boundary EC makes an angle of 58° with CD. Calculate the magnitude of angle CED, and check the result by drawing a plan of the field to a scale 1 in.=40 yds.

Area

A plane surface is such that if any two points be taken in it, the straight line joining them lies wholly in the plane surface.

Examples of horizontal plane surfaces are: a room floor, the surface of still water, the top of a table.

Examples of vertical plane surfaces are: the walls of a room, the upright sides of a box, the back of an upright cupboard.

EXERCISE 79

1. On gummed paper draw two squares having sides of 1 in. and 1 cm. respectively. Cut them out, fix them in the notebook, and label them 1 sq. in. and 1 sq. cm. respectively.

2. Construct on squared paper a square on a base of 3 ins., and show that

(i) It has 4 equal sides and 4 equal angles, each angle being a right angle.

(ii) The diagonals bisect each other and form 4 separate right angles at the point of bisection. Note that this point of bisection is the centre of the circumscribing circle.

(iii) Each diagonal bisects two opposite angles of the square.

3. Construct on squared paper a rectangle 5 ins. base, 3 ins. altitude. Compare it with the square and show that

(i) There are two pairs of opposite sides which are equal, and there are 4 equal angles, each being a right angle.

(ii) The diagonals bisect each other, but do not form 4 separate right angles at the point of bisection. Note that the point of bisection is the centre of the circumscribing circle.

4. (i) Find by drawing and by calculation the areas of squares having bases of : $1\frac{1}{2}$ in., $2\frac{1}{2}$ ins., $3\frac{1}{2}$ ins.

(ii) Find by calculation the areas of squares having bases of $1\frac{1}{2}$ in., $2\frac{1}{2}$ ins., $3\frac{1}{2}$ ins, . . . $11\frac{1}{2}$ ins.

Set out the results thus : $1\frac{1}{2}\times1\frac{1}{2}=2\frac{1}{4}$.
$2\frac{1}{2}\times2\frac{1}{2}=6\frac{1}{4}$.

5. Write a rule for squaring such numbers as : $1\frac{1}{2}$, $2\frac{1}{2}$, $3\frac{1}{2}$, etc.

6. Write answers only to the following :—

$7\frac{1}{2}$ lbs. of biscuits at $7\frac{1}{2}$d. per lb., $8\frac{1}{2}$ lbs. of plums at $8\frac{1}{2}$d. per lb., $(35)^2$, $(4{\cdot}5)^2$, $(6{\cdot}5)^2$, $(85)^2$.

7. Draw on squared paper the following rectangles :—

1 in. by 1 in., $\frac{1}{2}$ in. by 2 ins., $\frac{1}{4}$ in. by 4 ins. What is the

area of each rectangle ? Note that an area of 1 sq. in. may be any shape, but always occupies a definite amount of space.

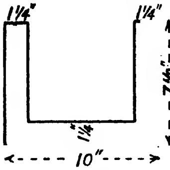


FIG. 54.

8. (i) Find the area of the floor of a square room, one side of which measures 19 ft. 6 ins.

(ii) Find the area of a rectangular field, the length of which is 40 yds. 2 ft., and the breadth 21 yds.

9. How many square feet are contained in the four walls of a room 12 ft. long, 10 ft. wide, and 10 ft. high, deducting 100 sq. ft. for the area of door and window ?

10. Find the area of the rolled channel bar section shown in fig. 54.

EXERCISE 80

1. Draw three separate square inches and cut them out. Cut each one in a different way, and rearrange the pieces to show that the amount of surface in a square inch may be any shape whatever.

2. There are 36 boards, each 9 ft. 6 ins. $\times$ 9 ins. When placed together they cover a floor space 8 ft. wide. How long is the space ?

3. A wooden fence is 80 ft. long and 6 ft. high. It takes 1 lb. of paint to cover 4 sq. ft. How much paint is used in giving the whole fence two coats of paint on both sides ?

4. What is the surface area of the outside of a box 4 ft. $\times$ 6 ft. $\times$ $3\frac{1}{2}$ ft. when the lid is removed ?

5. Show by diagram the maximum number of squares of side $\frac{1}{2}$ in. which can be cut from a rectangular sheet $4\frac{1}{2}$ ins. $\times$ 2 ins.

6. If the perimeter of a square is 30 cm., show how to construct the square. What is its area ?

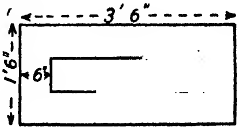


FIG. 55.

7. Copy fig. 55 to a scale of 1″ = 1 ft. Find by calculation the area of the border, and test your result by your copy.

8. Make scale drawings of the different rectangular faces

of a brick 9 ins. $\times 4\frac{1}{2}$ ins. $\times 3$ ins. From your drawings find the length of thread required to wrap round the brick in three different directions.

9. A square and a rectangle have each a perimeter of 1 ft. Construct them and show the difference in their areas. How many different rectangles fulfil this condition ?

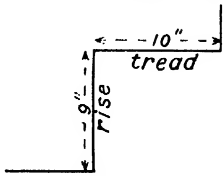


FIG. 56.

10. Show by a diagram the surface of a flight of 4 steps each having a 9-in. rise and a 10-in. tread ; the width of the steps being 2 ft. 6 ins. (see fig. 56).

Square Root

We have already learnt that $5^2=5\times5$; $5^3=5\times5\times5$, and so on. Read again p. 55.

Learn the second powers of the numbers from 1 to 20. Beginning at 13, these are :

$13^2=169$	$17^2=289$
$14^2=196$	$18^2=324$
$15^2=225$	$19^2=361$
$16^2=256$	$20^2=400$.

Learn the square roots of 400, 361, 324, etc.

When the square root of a number is not obvious by inspection, it may often be obtained by factorisation.

Example.—Find the square root of 7056.

Factorise thus :

2	7056
2	3528
2	1764
2	882
3	441
3	147
7	49
	7

$$\text{Factors}=2\times2\times2\times2\times3\times3\times7\times7$$
$$=2^2\times2^2\times3^2\times7^2$$
$$\therefore \text{Square Root}=2\times2\times3\times7$$
$$=\underline{84}.$$

Note.—The root of a fraction is the root of its numerator over the root of its denominator.

EXERCISE 81

1. Find by inspection the square root of
(i) 10,000, (ii) 289, (iii) 400, (iv) 225, (v) 256, (vi) 169, (vii) 196.

2. Find by factorisation the square root of
(i) 14,161, (ii) 72,900, (iii) 5625, (iv) 15,876, (v) $90\frac{1}{4}$.

3. The square of a certain number is equal to the square root of 20,736. What is the number?

4. A rectangular board 5 ft. 4 ins. by 4 ins. is cut into pieces which are fitted together to form a square. What is the base of the square?

5. The area of a draught board is 144 sq. ins. If the board is made up of 64 squares, what is the length of the base of each square?

6. The edge of a square tile is 4 ins. How many such tiles will just cover a square table top with a side 2 ft. 8 ins.?

7. What are the square roots of ·01, $6\frac{1}{4}$, 1, $2\frac{1}{4}$, ·25, 2500?

8. Find the squares of 1·5, 15, $1\frac{1}{2}$, 150, ·3.

9. The perimeter of a square is 2 yds. 2 ft. 4 ins.; find its area.

10. What is the difference in area between eight square inches and an eight-inch square?

Area of a Triangle

Draw on squared paper a rectangle 4 ins. by 3 ins. Letter it as shown in fig. 57, ABCD. In DC, take any point E and join EA and EB, thus forming the triangle EAB.

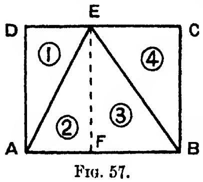


FIG. 57.

Now draw EF (very faintly) perpendicular to AB. Cutting out the four triangles numbered we find that triangle (1) is just equal in area to triangle (2), and that triangle (3) is also just equal in area to triangle (4).

Put them together again as in the figure, and consider this:

(i) The area of the rectangle ABCD $=$ AB $\times$ BC $=$ base $\times$ altitude.

(ii) The area of the triangle EAB $= \frac{1}{2}$ the area of the rectangle ABCD $= \frac{1}{2}$(AB $\times$ BC) $= \frac{1}{2}$(base $\times$ altitude).

We may try as many cases as we like, and we always find that the

$$\text{Area of a triangle} = \tfrac{1}{2}\ \text{base} \times \text{altitude} = \frac{\text{base} \times \text{altitude}}{2}.$$

Exercise 82

1. On squared paper draw the following triangles, each with an altitude of 1 in. and a base $1\frac{1}{2}$ in.

(i) $\triangle ABC$ having $\angle ABC=60°$.
(ii) $\triangle ABC$ „ $\angle ABC=90°$.
(iii) $\triangle ABC$ „ $\angle ABC=120°$.

In each case count the number of small squares in each triangle (omit squares less than half squares, and count parts of squares greater than half squares as whole ones). What do you observe?

2. The perimeter of an equilateral triangle is 6 ins. Draw the triangle and calculate its area.

3. Construct a triangle having a base of $3\frac{1}{4}$ ins. and an area of 7 sq. ins. How many different solutions of this problem are possible?

4. Find the area of a triangular brass plate having a base of 20·5 dm. and an altitude of 15·4 dm.

5. Find the area of the gable end of the house represented in fig. 58.

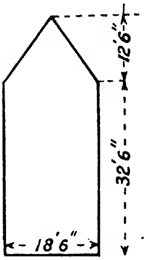


Fig 58.

6. Find the total surface areas of the triangular faces of a square pyramid if one edge of the base measures 5·8 cm. and the slant height is 7·5 cm.

7. Construct an equilateral triangle on a base of $2\frac{1}{2}$ ins., and on the same base construct a rectangle having an area equal to that of the triangle.

8. An isosceles triangle has an area of 10·5 sq. cm. and a base of 3·5 cm. Draw the triangle, and on the same base construct a right-angled triangle having the same area as the isosceles triangle.

9. A square metal plate is 6 ins. long, and a triangle having its base and altitude each equal to half the side of the square is cut out of the square. Find the area of the metal remaining.

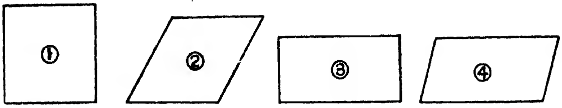
10. The span of a roof is 30 ft. and the rise is 13 ft. Find the area of a triangular cross section of the roof.

Parallelograms

A Parallelogram is a four-sided figure which has each pair of its opposite sides parallel.

There are four kinds of parallelograms, namely :

(1) Square ; (2) Rhombus ; (3) Rectangle ; (4) Rhomboid, as illustrated in fig. 59.



Square. Rhombus. Rectangle. Rhomboid.

Parallelograms.

FIG. 59.

A Rhombus has four equal sides, but its angles are not right angles. If a square were pivoted at its angular points and then pushed out of the shape of a square, it would form a rhombus.

A Rhomboid has its opposite sides equal, but its angles are not right angles. If a rectangle were pivoted at its angular points and then pushed out of the shape of a rectangle, it would form a rhomboid.

All parallelograms have (i) Opposite sides parallel and equal.
(ii) Opposite angles equal.
(iii) Diagonals bisecting each other.

Draw a parallelogram, as ABCD in fig. 60. Produce the line CD and draw the perpendiculars AP and BQ as shown. Cut off the triangles APD and BQC and show that they are equal in area. It is evident then that the area of the parallelogram ABCD= the area of the rectangle ABQP = **Base × Altitude.**

The areas of all parallelograms (square, rectangle, rhombus, rhomboid) are found in the same way, namely : **Base × Altitude.**

EXERCISE 83

1. If, in the fig. 60, AB=5·3 cm., and BQ=3·5 cm., find the area of the parallelogram ABCD. Write down from this result the area of the rectangle ABQP.

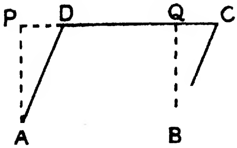


FIG. 60.

2. A field is in the form of a parallelogram, having one of its sides 35 yds. 2 ft. long. If its area is 749 sq. yds., what is the perpendicular distance between the two sides having lengths of 35 yds. 2 ft. ?

3. A sheet of tin is in the form of a parallelogram and has an area equal to that of a square of 10·5 ins. side. If one side of the sheet of tin is $12\frac{1}{4}$ ins. long, find the perpendicular distance between this side and the opposite side.

4. Find the areas of the following parallelograms :—

(i) Base, 3 yds. 27 ins. ; altitude, $2\frac{1}{2}$ yds.
(ii) Base, 7·8 ins ; altitude, 5·5 ins.

5. In fig. 61, find

(i) Area of rectangle FECA.
(ii) ,, triangles AFB and CED.
(iii) ,, parallelogram FEDB.
(iv) ,, figure FECB.

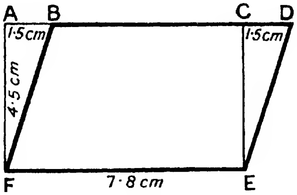


FIG. 61.

6. Draw the four different kinds of parallelograms and find by measurement and calculation the exact areas of the figures drawn.

7. State the common properties of (i) a square and a rhombus, (ii) a rectangle and a rhomboid.

8. State the difference between (i) a square and a rhomboid, (ii) a square and a rectangle, (iii) a rectangle and a rhomboid, (iv) a rhombus and a rhomboid.

9. Construct a square decimetre full size and then draw a rhomboid having the same area. Prove that your two figures are equal in area, in a way similar to the proof given on p. 100.

10. Draw the four kinds of parallelograms, each having an area of 9 sq. ins.

Area of a Circle

Draw on gummed paper a circle having a radius of 2 ins., and, as in fig. 62, divide it into 16 equal sectors. Cut out these

sectors and arrange them in the form of a parallelogram, as in fig. 63.

$$AB = \frac{1}{2} \text{ the circumference} = \pi r \text{ ; altitude} = \text{radius} = r.$$
$$\therefore \text{ area of circle} = \pi r \times r$$
$$= \pi rr = \pi r^2.$$

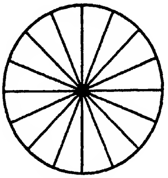


Fig. 62.

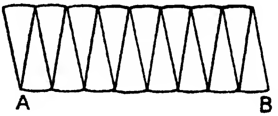


Fig. 63.

Exercise 84

1. Calculate the area of a circle which has a radius of 3·4 ins. ($\pi = 3{\cdot}14$). Check the result by constructing such a circle on squared paper and counting the squares.

2. Construct a rectangle 2·4 ins. by 1·8 ins. Draw the circumscribing circle, and having measured its radius, find its area.

3. Calculate the area over which a horse can graze if it is tethered to a pole by a rope 5 ft. in length.

4. Circular holes are punched in a square metal plate, as shown in fig. 64. What is the area of the metal remaining?

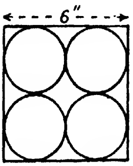


Fig. 64.

5. Six halfpennies are placed flat on a table. What area do they cover? First determine the diameter of a halfpenny.

6. The external diameter of a washer is 1·75 in., and the internal diameter is ·75 in. What is the area of one of the flat surfaces of the metal?

7. A rectangular field is 38 yds. × 64 yds., and on it six conical tents having bases 4 yds. in diameter are pitched. Find the surface of the field not under cover.

8. How many plants may be planted in a circular garden plot, diameter 4 ft. 8 ins., if each plant requires a space of 44 sq. ins.?

9. The maximum distance across a round table top is 4 ft. Find the cost of polishing the top at 6d. per square foot.

10. Supply the particulars to complete the following table ($\pi = 3 \cdot 14$):—

	Radius.	Diameter.	Circumference.	Area.
1	5 ft.	..	..	..
2	..	7 ins.	..	..
3	2·5 cm.	..	..	..
4	..	..	251·2 ins.	..
5	..	4·2 m.	..	..
6	8 yds. 2 ft.	..	..	..
7	1·8 m.	..	..	..
8	..	..	84 yds.	..
9	..	3·8 ft.	..	..
10	..	..	2198 cm.	..

Development of a Solid

A solid may be constructed by placing its faces in correct position, *e.g.* six squares can be arranged to form a cube. The plane figure representing the total area of a solid is known as the development of the solid.

Figs. 65, 66, 67 show the development of the cube, the cuboid, and the cylinder.

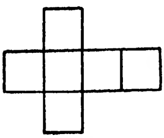


FIG. 65.

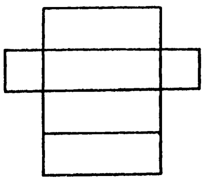


FIG. 66.

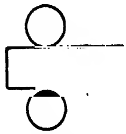


FIG. 67.

EXERCISE 85

1. Using squared paper, draw the development of a cuboid 1·5 ins. × 1·5 ins. × 3 ins. Allow in your drawing for the necessary flanges, and then cut out and fold up to form a solid.

2. Using squared paper, draw the development of a cube having an edge 1·3 in. long. What is its total surface area ?

3. Show by a diagram the amount of cardboard wasted in making a rectangular prism ·7 in. × ·8 in. × 1·3 in. from a sheet of cardboard 4 ins. × 3·5 ins.

4. Draw a figure to represent the total surface area of a cylinder $1\frac{1}{2}$ ins. in diameter and $2\frac{3}{4}$ ins. in length.

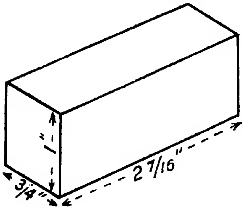


FIG. 68.

5. Draw the development of the solid shown in fig. 68.

6. A block of wood is a rectangular prism a ins. × b ins. × c ins. Draw a hand sketch of its development, and find its total surface area and the total length of its edges.

7. Draw the development of a square prism 1·3 in. × 1·3 in. × 2·4 in. How many of its edges are equal to each other ?

8. Draw a plane figure which could be cut out and folded to form a rectangular prism 2 ins. × 1 in. × $1\frac{1}{2}$ in. Find its total surface area.

9. A cardboard rectangular prism is 5·3 ins. × 4·7 ins. × 3·4 ins., and all its edges are bound with adhesive tape. Find the length of tape used for the purpose.

10. Draw a freehand sketch of a rectangular block of wood 6 ins. × 6 ins. × 18 ins. Name the solid and find its surface area.

11. How much canvas is necessary to cover a box 3 ft. × $1\frac{1}{4}$ ft. × $2\frac{1}{2}$ ft. ?

12. What length of wire is required to construct the framework of a box 10 ins. × 8 ins. × 2·5 ins. ?

13. A roller is 10 ins. long and 2 ins. diameter, what area will it cover if it makes 5 revolutions ?

14. The base of a rectangular tank is x yds. by z yds., and it contains water to a depth of p yds. Find the total surface wetted.

Volume

The standard for expressing volume is the space occupied by a cube of known volume, *e.g.* a solid having a volume of 5 cu. ft. occupies the same space as five cubes, each having a volume of 1 cu. ft. ; a solid having a volume of 20 c.c. occupies the same space as 20 cubes, each having a volume of 1 c.c.

The volume of all right prisms can be calculated thus:

Volume = Area of Base × Height of Prism.

Note that the base of a prism is the plane figure on which the solid is built up, *e.g.* the base of a cube is a square, the base of a cylinder is a circle.

Exercise 86

1. A block of clay is 18 ins. × 4 ins. × 3 ins.; into how many cubes of half-inch side may it be modelled?

2. What is the capacity of a rectangular box which exactly holds 12 bricks each 9 ins. × $4\frac{1}{2}$ ins. × 3 ins.?

3. What is the cubical content of a gasholder on a circular base having a radius of 12 ft. and a height of 16 yds.?

4. A block of wood is cubical, and its volume is $27a^3$ cu. ins. Find (*a*) the length of one side, (*b*) the total length of its edges, (*c*) its surface area.

5. Calculate the area of the cross section of a cylindrical tree trunk of volume $58\frac{1}{2}$ cu. ft. and height 9 ft. 9 ins.

6. A rectangular tank 2 ft. × 2 ft. × 4 ft. $1\frac{1}{2}$ ins. is filled with water. How many times may a cylindrical bucket of radius 6 ins. and depth 18 ins. be filled from the tank?

7. A cylindrical flour bin has a diameter 2 ft. 8 ins. and is 30 ins. deep. Find to the nearest pound the weight of flour it contains if 1 cu. ft. of flour weighs 12 lbs.

8. Complete the following table of particulars of rectangular prisms:—

Base.	Altitude.	Height.	Volume.
5·8 cm.	6·2 cm.	4·6 cm.	..
..	5 ins.	5 ins.	39·75 cu. ins.
7·9 m.	..	2·0 m.	58·8 cu. m.
4 ft. 3 ins.	5 ft. 6 ins.	..	80 cu. ft.
$6\frac{1}{4}$ yds.	8 ft.	$7\frac{1}{2}$ ft.	..
17·2 ins.	5 ins.	2·25 ins.	..

9. In a beam 14 ft. 6 ins. long, having a cross-section area of $4\frac{3}{4}$ sq. ft., there are three circular holes bored. If each of the holes is 4 ins. in diameter and $1\frac{1}{2}$ ft. deep, find the volume of material in the beam.

10. A tank is 5·8 metres long and 3·5 metres broad. To what depth is it filled when it holds 31,000 litres of water ?

11. To mould an iron bar $15\frac{1}{2}$ ft. long and $7\frac{1}{4}$ ft. wide costs £12, 16s. 0d. If the rate is 6d. per cu. ft., find the thickness of the bar.

12. If a cubic foot of aluminium weighs 160 lbs., find the area of a sheet of aluminium $1\frac{1}{2}$ ins. thick which weighs 32 lbs.

13. A rectangular room is 18 ft. in height. Find the area of its floor space if it contains 3456 cu. ft. of air.

14. The area of a field is 2 acres, and rain falls on it to a depth of $1\frac{1}{2}$ ins. Find in tons the weight of water on this field if 1 cu. ft. of water weighs 1000 ozs.

15. A brick 9 ins. $\times 4\frac{1}{2}$ ins. $\times 3$ ins. is dropped into water in a cylindrical bucket. If the diameter of the bucket is 1 ft. 2 ins., by how much will the water level rise ?

16. The outside dimensions of an open box are $14\frac{1}{2}$ ins. $\times 11\frac{1}{2}$ ins. $\times 8\frac{3}{4}$ ins. If the box is made from wood $\frac{1}{2}$ in. thick, find its capacity.

Summary of Formulæ

Square

Perimeter = sum of sides = one side × 4.
Area = (one side)^2.
Side = $\sqrt{(\text{area})}$.

Rectangle

Perimeter = sum of sides = 2 (length + breadth).
Area = base × altitude = length × breadth.

Parallelogram

Area = base × altitude.

Triangle

Perimeter = sum of sides.
Area = $\frac{1}{2}$(base × altitude).

Circle

Circumference = π × diameter = 2π × radius.
$= \pi d = 2\pi r$.
Area = πr^2.

Cube

Surface = (one edge)$^2 \times 6$
Volume = (one edge)^3.

Prism

Volume = area of base × altitude.

Cylinder

Area of curved surface = $2\pi r \times$ altitude.

Volume = $\pi r^2 \times$ altitude.

Exercise 87

Miscellaneous Problems

1. Measure the distance from X to Y : (i) in cm. and mm. ; (ii) in ins. and tenths of an inch.

X ____ Y

Calculate from your results the number of cm. in an inch. Give answer to two decimal places.

2. Find the perimeter of the doorway shown in fig. 69.

3. A rectangular room 27 yds. 2 ft. by 12 yds. 1 ft. is to be covered with blocks having a surface 8 ins. by 4 ins. How many blocks are required ?

4. How many square feet are contained in the four walls of a rectangular room 12 ft. 6 ins. by 11 ft. 6 ins., and 10 ft. 6 ins. in height ?

5. How many yards of carpet 2 ft. 9 ins. wide will be requirèd to cover the floor of a room 16 ft. 6 ins. long and 11 ft. wide ?

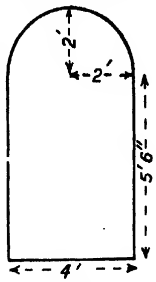


Fig. 69.

6. Find the cost, at 6d. a sq. ft., of polishing the top of a circular column having a diameter of 3 ft. 6 ins.

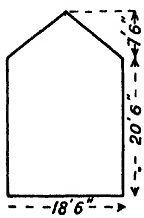


Fig. 70.

7. Find the cost, at 1s. 9d. a square yard, of slating the end of the house shown in fig. 70.

8. A rectangular prism is x ins. long, y ins. wide, and z ins. high. Write (*a*) the total length of its edges, (*b*) the total area of its faces, (*c*) its volume.

9. Find the volume of

(i) A rectangular prism 10·5 cm. long, 8·5 cm. wide, and 9·5 cm. high.

(ii) A square prism having a base of 12·5 cm. side and a height of 20 cm.

10. A cistern measures 7·8 m. by 5·6 m. by 4·5 m. Find in Kg. the weight of the water the cistern will hold. (1 c. dm. of water weighs 1 Kg.; 1 c. metre =1000 c. dm.)

11. How many cubic feet of air space is allowed per pupil when a class-room 27 ft. by 24 ft. by 20 ft. accommodates 30 pupils ?

12. Find the area of a rectangular playground $50x$ ft. long and $20y$ ft. wide.

13. Write the perimeters of (i) a triangle whose sides are $2x$ ins., $3y$ ins., and $5z$ ins.; (ii) an equilateral triangle on a base of $3x$ ins.; (iii) a square on a base of $2x$ ins.; (iv) a rectangle $3x$ ins. by $4x$ ins.; (v) a circle having a diameter of $2x$ ins.

14. Write the area of (i) a triangle having a base x ins. long and an altitude of y ins.; (ii) a square on a base $3x$ ins. long; (iii) a rectangle $4x$ ins. by 5 ins.; (iv) a circle having a diameter of $3x$ ins.

15. Write the volume of (i) a cube having one edge $2x$ ins. long; (ii) a rectangular prism $2x$ ins. by $3x$ ins. by $4x$ ins.

16. Write the sides of the squares having areas of $36r^2$, x^2y^2, $49y^2$, $64z^4$, $81a^2b^2$.

17. A central hall is 24 yds. long and 16 yds. wide. Find the cost of covering it with wooden blocks 8 ins. by 4 ins., at 4s. 6d. a dozen.

18. Two towns are shown on a map as being $5\frac{1}{2}$ ins. apart. If the scale is $\frac{1}{63360}$, what is the actual distance from one town to the other ?

19. A field is in the shape of a parallelogram. The length of one side is $84\frac{1}{2}$ yds., and the altitude is $34\frac{1}{2}$ yds. Find the area in square yards.

20. One of the driving wheels of a locomotive is 6 ft. in diameter. If this wheel makes 200 revolutions per minute, find the speed of the train, (i) in miles per hour, and (ii) in feet per minute. ($\pi=3\frac{1}{7}$.)

21. A perambulator wheel has a diameter of $1\frac{1}{2}$ ft. How many revolutions does it make in travelling 60 yds. ? ($\pi=\frac{22}{7}$.)

22. Find the area of the top of a desk 30·5 cm. by 12·4 cm.

23. Find the cost, at $5\frac{1}{2}$d. per foot, of the skirting board round a room 21 ft. 6 ins. by 28 ft. 6 ins., if 12 ft. are deducted for the fireplace and the door.

24. Find the cost of colouring the four walls of a room, 18 ft. 6 ins. long, 16 ft. 6 ins. wide, and 12 ft. high, at 6d. a square yard.

25. When a train is travelling at the rate of 44 km. per hour, how often does a carriage wheel of 1 metre diameter revolve per minute? ($\pi = 3\frac{1}{7}$.)

26. Find the area of a rectangular garden plot 18 ft. 6 ins. long by 15 ft. 6 ins. wide. Give the answer in square yards, square feet, and square inches.

27. A bowling-green measures 30 yds. by 24 yds. Find the area of a path 6 ft. wide all round it.

28. Find the cost, at $3\frac{1}{2}$d. per sq. yd., of decorating the walls of a room 22 ft. 6 ins. by 20 ft. 6 ins., and 12 ft. 6 ins. high. Deduct 100 sq. ft. for doors, windows, and fireplace.

29. A circular racing-track is $\frac{1}{2}$ a mile in length. Find the diameter. ($\pi = \frac{22}{7}$.)

30. Fig. 71 represents a plot of ground 100 yds. square. In the middle is a circular lake having a diameter of 20 yds. Find the area of the remaining ground in square yards. ($\pi = 3{\cdot}14$.)

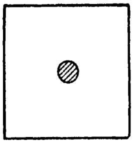


FIG. 71.

31. Find the cost, at 2s. per sq. in., of a circular piece of bronze 1 ft. 6 ins. in diameter. ($\pi = 3{\cdot}14$.) (Give answer to the nearest penny.)

32. How many hoops, each having a diameter of 2 ft. 6 ins., can be made from 24 yds. of hoop-iron? What length remains? ($\pi = \frac{22}{7}$.)

TEST PAPERS

A

1. How often could ·238 be successively subtracted from 8·05623, and what would the remainder be ?

2. There are three lighthouses in an estuary. One signals every 14 minutes ; one every 3 minutes ; and the third twice every 5 minutes. They signal together at 8 p.m. When will they next signal together ?

3. Express as £, s. d.

$$\frac{0{\cdot}171 \times {\cdot}0000654}{{\cdot}00001 \times 17{\cdot}1 \times 65{\cdot}4} \text{ of } £1000.$$

4. A can run 100 yds. in 11 secs., and B can run the same distance in $11\frac{1}{2}$ secs. How many yards start can A give B in 300 yds.?

5. A man bought 84 oranges at 3 for 2d., and 84 more at 4 for 2d. He mixed them and sold them at 7 for 4d. Find his actual gain or loss, and also his gain or loss per cent.

6. Find the interest on £276, 5s. 0d. for 8 months at $3\frac{3}{4}$ per cent. per annum.

7. A space is triangular in shape. Its base is 45 ft. 6 ins. long, and the perpendicular height from the base to the apex of the triangle is 12 ft. Find the cost of tiling the space at 8s. 6d. per square yard.

8. Find the value of

(i) $3^2+5^2-\sqrt{49}$

(ii) $\sqrt[3]{27}-\sqrt[4]{16}+\sqrt{81}$

(iii) $\sqrt{100}-\sqrt[3]{64}$

(iv) $(7-2)^2+\sqrt{121}$

What is the value of $\sqrt{m^2+2mn+n^2}$ when $m=5$ and $n=4$?

9. When A represents $5x+3$ and B represents $3x$, find the value of $2AB^2$.

10. (*a*) Make a hand sketch of a match-box showing its dimensions.
    (*b*) Draw the development of the match-box.
    (*c*) Calculate its outer surface area.

B

1. George correctly divided a certain number by 37 and obtained 36 as a remainder. Mary correctly divided the same number by 74 and obtained a remainder which was not 36. What was her remainder ? Say why.

2. Simplify $\dfrac{2\frac{4}{5}-1\frac{1}{2}+9\frac{1}{11}}{4\frac{1}{5}-2\frac{1}{4}+13\frac{4}{11}}$.

3. How many pieces, each ·07 in. long, may be cut from 10·7 yds. of wire ? How much is left ?

4. A fruiterer makes a contract with a carrier to make 300 journeys each of $8\frac{1}{4}$ miles for £165. If the carrier were paid separately for each journey the charge would be 1s. $7\frac{1}{2}$d. per mile. How much does the fruiterer save by his contract ?

5. The following is a short way of finding the cost of 1000 articles when the cost of one is given :—

"Reduce the cost of one article to farthings. Then reckon as many sovereigns and ten times as many pence as there are farthings in the cost of one."

Use the rule to find the cost of

(a) 1000 articles at $5\frac{3}{4}$d. each.
(b) 1000 ,, 1s. $4\frac{1}{2}$d. each.
(c) 2000 ,, 2s. $7\frac{1}{2}$d. ,,

6. How much does a man lose if he borrows £185 at $4\frac{1}{4}$ per cent. per annum for 4 years when he might have borrowed it at $3\frac{1}{2}$ per cent. per annum ?

7. It costs £16, 14s. 10d. to paint the walls of a room at 7d. per sq. ft. If the room is $18\frac{1}{2}$ ft. long, 14 ft. wide, and $11\frac{1}{4}$ ft. high, how many square feet do the doors and windows occupy ?

8. When $a=\frac{1}{4}$, find the value of $a+a^2+a^3$.

9. In the equation $4x-1=2y$, find the values of y when $x=1$, $x=2$, $x=4$.

10. Draw a line 3·7 ins. long, and divide it in the ratio 3 : 5.

C

1. A man tried to run 10 miles in 1 hr. 5 mins. He ran the first mile in 5 mins. 20 secs., and the second mile in 5 mins. 40 secs. If he had run the remaining 8 miles at the average speed of the first two, how much time would he have had to spare ?

2. There are only four numbers between 10 and 100 which have 6 as their H.C.F., and 630 as their L.C.M. Find them, and explain your method.

3. Share £128·25 between two persons in such a way that the first receives as many half-crowns as the second receives florins.

4. A man and his son work together. For every sovereign the

man earns the son earns 3s. 3d. If together they earn £325, 10s. 0d., how much does each earn ?

5. A boy has five times as much money as his sister. He gives her 1s. 8d., and then they have equal amounts. Find how much each had at first. By how much per cent. was the girl's money increased when the boy shared his money with her ?

6. On 20th December 1924 the price of Egyptian twist cotton was $28\frac{1}{8}$d. per lb. ; the price of silver on the same day was $35\frac{7}{8}$d. per oz. Find by the shortest method (i) the cost of 10,000 lbs. of Egyptian twist cotton, (ii) the cost of 20,000 ozs. of silver.

7. If it costs £24, 10s. 0d. to fence a square plot, using fencing at 3s. 6d. a yard, find the cost of covering it with turf at 2d. per sq. ft.

8. Find the value of (i) $2x-(7x-8y-8x)+6x+(4y-3y)$.

(ii) $-\{x-[2x-(2x-x)]\}-3[x+(3x-2x)]$.

9. A train travels x yds. in y min. How far does it travel in an hour ? How long does it take to travel 100 miles ?

10. Using squared paper, show that $(2{\cdot}7 \text{ ins.})^2=7{\cdot}29$ sq. ins.

D

1. One man had an income during a certain year of £274, and another man had an income of £300. The next year both men increased their incomes by $12\frac{1}{2}$ per cent. Find the actual difference in their incomes for the second year.

2. A reservoir is $\frac{5}{8}$ full of water on Sunday evening : $\frac{1}{25}$ of its contents is drawn off on Monday, and 4000 galls. on Tuesday. On Tuesday night it was half empty. How much will it hold ?

3. Which is the greater, and by how much : 275 per cent. of £6, 18s. 6d. or 305 per cent. of £5, 11s. 8d. ?

4. In 1911 the population of a town was 60,000, and in 1921 the population had increased to 72,500. Find the increase per cent.

5. If 1000 grams $=2{\cdot}20462$ lbs., find the difference in lbs. between 100 kg. and 2 cwts.

6. How many threepences are there in £x, ys. zd. ? How many halfpence are there in $(2a-3b)$ half-crowns ?

7. A grocer mixes $6x$ lbs. of tea at 2s. per lb. with $10x$ lbs. at 2s. 6d. per lb. What should he charge for one pound of the mixture ?

8. Which is the greatest of the following :—

$$\frac{1}{37{\cdot}5},\quad \frac{1}{{\cdot}375},\quad \frac{1}{3{\cdot}75}.$$

How can this be told without working the sum ?

9. Find the area of glass in a window having a semicircular top, if the width of the window is 16 ins. and its total length 8 ft. 6 ins

10. Draw a rectangle 15 cm. by 4 cm. and shade 15 per cent. of it.

E

1. Draw a rectangle $3\frac{1}{2}$ ins. by $2\frac{1}{2}$ ins. What is its area in square inches?

Suppose an inch is equal to 2·54 cm. Find the area of the rectangle in square centimetres.

2. A space is triangular in shape. The base is 15 yds. 2 ft. long, and the perpendicular height is 5 yds. What is the area of the space in square yards, and what would be the cost of covering it with slates at 6s. 9d. per sq. yd.?

3. A can walk 100 yds. in 30 secs., and B can walk the same distance in $32\frac{1}{2}$ secs. How many yards start can A give B in 300 yds.?

4. What will be charged as simple interest on £58, 7s. 6d. for 73 days if the rate is $12\frac{1}{2}$ per cent. per annum?

5. A man advertises a cylindrical tank of diameter 2 ft. 4 ins. and height 5 ft. 6 ins. as being capable of containing 200 galls. Is his estimate correct? You may take π as $3\frac{1}{7}$ and a cubic foot as containing $6\frac{1}{4}$ galls.

6. I invest £1050 on 17th April and withdraw the principal and interest on 22nd November. How much do I withdraw if the bank pays simple interest at $3\frac{1}{2}$ per cent. per annum?

7. What quantity divided by $6yz$ gives a quotient $2y^2z^2$? When 4 times x is taken from 13 the remainder is 1. Find x.

8. A circular pond is 28 ft. across. If it is covered with ice to a depth of $1\frac{1}{2}$ ins., find the volume of ice on the surface of the pond.

9. If 18 lbs. of butter cost £2, 5s. 0d., draw a graph to illustrate this and also the cost of $7\frac{1}{2}$ lbs., $4\frac{3}{4}$ lbs., 8 lbs.

10. Show by diagram that $x(m+n)=mx+nx$.

F

1. A grower picks 30 cwts. of pears and packs them in baskets holding $8\frac{3}{4}$ lbs. each. If each basket sells for 2s. $8\frac{1}{2}$d., how much money does he receive?

2. If 18 men dig over a field of 12 acres in 14 days, how long will it take the same men to dig over a field of $18\frac{1}{2}$ acres?

3. How much money must I return on 8th May 1928 if I borrowed £740 on the 1st January 1928 and agreed to pay $4\frac{1}{2}$ per cent. per annum as interest on the loan?

4. The ordinary return fare from Manchester to Bristol is 30s. 4d. The excursion rate is single fare ($\frac{1}{2}$ of return fare) and a third. If 602 persons travel at ordinary rate and 849 travel at excursion rate, how much less does the railway company receive than if all travelled at ordinary rate?

5. A greengrocer was fined for having a 7-lb. weight $17\frac{1}{2}$ drams light (1 oz. = 16 drams). Show that the weight was about 1 per cent. too light.

6. In 1905 the Government paid interest on six hundred and fifty million pounds at $2\frac{1}{2}$ per cent. per annum. Find the total interest paid in a year.

7. The sum of three consecutive whole numbers is 840. Find the numbers.

8. Given that $K = u^2 + 2as$, find K when $u = 39$, $a = 15{\cdot}8$, $s = 203{\cdot}7$.

9. Whilst a man rows across a circular pond which is 504 yds. across, his dog runs round the edge of the pond to meet him. How much farther does the dog travel than the man ?

10. Make a hand sketch showing the dimensions of a cube having sides 2 cm. long. Calculate (*a*) the number of edges; (*b*) total length of the edges; (*c*) total surface area; (*d*) its volume in c.c.

G

1. Divide one million four thousand and eighty-nine by three hundred and ninety-seven.

2. The postage rates for letters are "weight not exceeding 3 ozs., 2d.; for each additional ounce or fraction of an ounce, $\frac{1}{2}$d." What is the postage on a letter weighing $8\frac{3}{4}$ ozs. ? Also, what is the greatest weight that may be sent for $7\frac{1}{2}$d. ?

3. Write answers only to the following :—

(*a*) Cost of 3 dozen eggs at $3\frac{1}{2}$d. each.
(*b*) Cost of 1 yd. cloth if 12 yds. cost £4, 1s. 0d.
(*c*) £70, 10s. 6d. less 10 per cent.
(*d*) 732×25.

4. Find the value of $\dfrac{3{\cdot}25 + {\cdot}75}{2{\cdot}75 \div {\cdot}55}$.

5. How much zinc must be mixed with $2\frac{1}{2}$ cwts. of tin to make an alloy contain 25 per cent. zinc ?

6. An article was marked at £7, 10s. 0d. and sold for £6, 7s. 6d. What was the rate per cent. of the discount ?

7. Write an expression for the area of the iron plate shown in fig. 72.

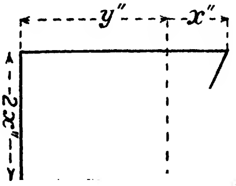


FIG. 72.

8. Divide £197 among X, Y, and Z, so that X may have £15 more than Y and Y three times as much as Z.

9. A man has a rectangular garden having an area of 375 sq. yds. Its length is 75 ft. Find the cost of fencing the four sides at 10d. a foot.

10. Draw a rectangle 3 ft. by $1\frac{1}{2}$ ft. to a scale of 1 in. to 1 ft. On the longer side of the rectangle draw a triangle having the same area as the rectangle.

H

1. A train leaves Manchester at 2.32 p.m. and runs 184·5 miles to London, where it arrives at 6.52 p.m. What is its average speed in miles per hour?

2. Find the value of (i) $8\frac{9}{20}+2\frac{3}{4}-3\frac{4}{5}+9\frac{5}{12}+6\frac{1}{2}$.
(ii) $6\frac{5}{6}+1\frac{3}{4}\div\frac{1}{2}$.

3. Tea can be bought either at 2s. per lb. or at $2\frac{1}{2}$d. per ounce. If I use 2 ozs. per week, what do I lose in a year by buying at the second price?

4. Find the total cost of the following. Write down the items in this form :—

			£ s. d.
4 iron rods, each 13 ft. 6 ins. long .	@ $10\frac{1}{2}$d. per ft.	=	
4 iron plates, total weight 831 lbs. .	@ $\frac{1}{2}$d. per lb.	=	
4 lengths iron tube, each 8 ft. 6 ins. long	@ 1s. 5d. per ft.	=	
$1\frac{1}{2}$ ft. lead water pipe . . .	@ 1s. 3d. per ft.	=	
16 lbs. pipe nails	@ 2d. per lb.	=	
78 lbs. sheet lead	@ $7\frac{1}{2}$d. per lb.	=	
Plumber—58 hours . . .	@ 2s. 6d. per hr.	=	
Labourer—58 hours . . .	@ 1s. 10d. per hr.	=	
	Total cost . .	=	

Deduct $7\frac{1}{2}$ per cent. from the total cost.

5. A train is timed to arrive in Carlisle at 5.30 p.m. and in Glasgow at 8 p.m. It is half an hour late at Carlisle. What average speed must it attain throughout the rest of the journey to arrive in Glasgow in time, the distance being 100 miles?

Also—By how much must it exceed its ordinary average speed?

6. A rectangular field is 340 yds. long and 256 yds. wide. Find the perimeter in kilometres.

7. A girl has x nuts; she received 26 more and lost 43, and then she had 18 nuts. How many had she at first?

8. The ninth part of a number, together with twice the number, is equal to three times the number less 32. Find the number.

9. A cylindrical post is 8 ft. 8 ins. long and 18 ins. in diameter.

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